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DIPOLES IN DISSIPATIVE MEDIA

By

Ronald W. P. King

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Cruft Laboratory
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Dipoles in Dissipative Media*

by

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Abstract

The general problem of antennas in dissipative media is reviewed with reference to possible applications and types of antennas. The question of bare and insulated antennas is discussed with reference to the highly-conducting center-driven cylinder of small cross section. The delta-function generator as a convenient idealization of feeding by transmission lines is considered; the apparent difficulty of terminals short-circuited by the conducting medium is resolved.

The problem of determining the admittance, the distribution of current, and the electromagnetic field for a cylindrical antenna in a conducting dielectric is presented with emphasis on the dual requirements of reasonable accuracy and simplicity. Methods used to solve the integral equation for the current in antennas in air are reviewed, and the limitations of their results for the calculation of the complete electromagnetic field are pointed out.

An approximate method of solving the integral equation for the current in an antenna in a dissipative medium is described, and a simple, reasonably accurate solution is displayed. The specific evaluation of currents and impedances for half-wave dipoles and electrically short antennas is summarized. The separation of the current into a principal part for which the electromagnetic field may be evaluated in closed form, and a part that may be neglected in important special cases is explained.

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The complete electromagnetic field of a half-wave dipole in a dissipative medium is determined first in cylindrical coordinates, then in spheroidal coordinates. The nature of the wave fronts and the polarization of the electric field are considered fairly close to the antenna and at greater distances. Similar results for the electrically short antenna are also given. The power dissipated in finite parts of the medium is determined.

Introduction

The single antenna and arrays of antennas in a dissipative medium are analytically and experimentally interesting and offer difficult problems that have numerous applications. These include, for example, communicating from a submerged submarine or from a mine shaft, and carrying out geophysical explorations with antennas directly imbedded in the earth or lowered into air-filled holes. Of more recent interest is the use of antennas as probes in the study of ionized regions and plasma sheaths by means of rockets. In general, the regions involved do not consist of a single, homogeneous and isotropic medium. There may be two distinct layers such as the sea or the earth and the air above it; alternatively, as in the interior of the earth, or in a plasma sheath in the ionosphere, the medium may have continuously varying properties or it may be stratified in one way or another. Experimental and theoretical studies of bare and insulated antennas and arrays in and over dissipative and dielectric media that are isotropic, have continuously varying properties, or are stratified in a regular manner are in progress. However, this report is limited to a study of the properties of single antennas of moderate length in an infinite, homogeneous, isotropic, dissipative medium.

Investigations that relate to antennas in dissipative media may be separated into two groups: (a) those which are concerned only with the electromagnetic field at large distances from a source, so that in the interest of simplicity, the antenna may be idealized far beyond practical availability; (b) those that treat the circuit properties of the antenna itself as well as the field that it generates. The source used in the former group usually consists of a Hertzian dipole or infinitesimal doublet that is represented mathematically by a periodically varying electric moment concentrated at a point.

Physically, such a doublet may be visualized as made up of equal positive and negative charges oscillating back and forth in opposite directions along a line in the limit as the charges become infinite and the length of their path vanishes. In the second group are studies of thin cylindrical antennas and of biconical structures immersed in dissipative media. The antenna may be in direct contact with the conducting medium or it may be surrounded by an intervening layer of insulation. Note that a sufficiently short antenna is the physically realizable approximation of an infinitesimal doublet, insofar as the distant field is concerned.

An early study of the radiation of a Hertzian dipole immersed in a dissipative medium was made by C. T. Tai [1], who obtained the electromagnetic field and the Poynting vector in the well-known forms for doublets in air, but with the permittivity and propagation constants both complex instead of real. He noted that the total power transferred into a dissipative medium, as obtained from an integration of the normal component of the Poynting vector over a sphere with its center at the doublet, becomes infinite when the radius of the sphere is reduced to zero and concluded that "it is impossible to speak of the total power radiated by a Hertzian dipole when the latter is in direct contact with a dissipative medium." He then proceeded to analyze the doublet enclosed in an insulating sphere [2]. Actually, the power transferred across a spherical surface that encloses charges oscillating along a line is not obviously defined in the limit as the radius of the enclosing sphere vanishes. Indeed, it is shown in a later section that the power radiated by an electrically short dipole in a dissipative medium cannot be obtained by integrating the normal component of the Poynting vector over a spherical surface. It may be remarked in passing that the infinitesimal doublet has been used as an idealized source in numerous fairly recent studies of the electromagnetic field in a conducting half-space [3, 4, 5, 6], since the properties of a finite radiating system could in this way be avoided.

The integral equation and its formal solution for the current in a cylindrical antenna of finite length immersed in a dissipative medium was formulated by Tai [7] in a manner paralleling the analysis of King and Middleton [8] for an antenna in the air. The essential difference is that

the previously real permittivity and propagation constant have become complex, with a resulting complication of the kernel and of integrals that occur in the iteration. Owing to a lack of tabulated functions, Tai did not evaluate his formal solution. Indeed, the analytical difficulty associated with the infinite admittance of the delta-function generator--which was not well understood at the time--led him to drop further work on the cylinder and turn his attention to the more tractable problem of the insulated biconical antenna in a dissipative medium. Tai's work on the dipole has been extended somewhat by Macrakis [9], Harrison [10], and Harrison and Denton [11] who made approximate evaluations of the impedance; more recently King and Harrison [12], and King, Harrison, and Denton [13], have carried out complete analyses of the circuit properties respectively of the half-wave dipole and of the electrically short antenna in a dissipative medium. These studies are based on the approximate method proposed by King [14] for the solution of the integral equation for the current in a cylindrical antenna. The present investigation is directed to the determination of the circuit properties of a thin cylindrical antenna of moderate but arbitrary length and to a consideration of the complete electromagnetic fields generated by the currents in such antennas.

Review of the Theory of Cylindrical Antennas in Air

Since the analysis of the properties of an antenna in a dissipative medium is a considerable complication of the problem of the same antenna in a perfect dielectric, it is well to review briefly the extensive theory of the cylindrical antenna in air. A simple, physically realizable circuit consists of a cylindrical conductor, center-driven from a balanced two-wire transmission line, as shown in Fig. 1a. Since the currents in the antenna and in the line satisfy two simultaneous integral equations, their determination is a formidable problem. If the two conductors of the transmission line are very close together, the significant interaction of the line and the antenna is confined to a small region near their actual junction. For the line, this may be approximated by a reactive network of lumped elements characteristic of the line combined with the impedance Z_0 of the antenna also as a lumped element, as shown in Fig. 1b. From the point of view of the antenna, the driving field across the end of the feeding line, which is distributed over a short length of the antenna, may be

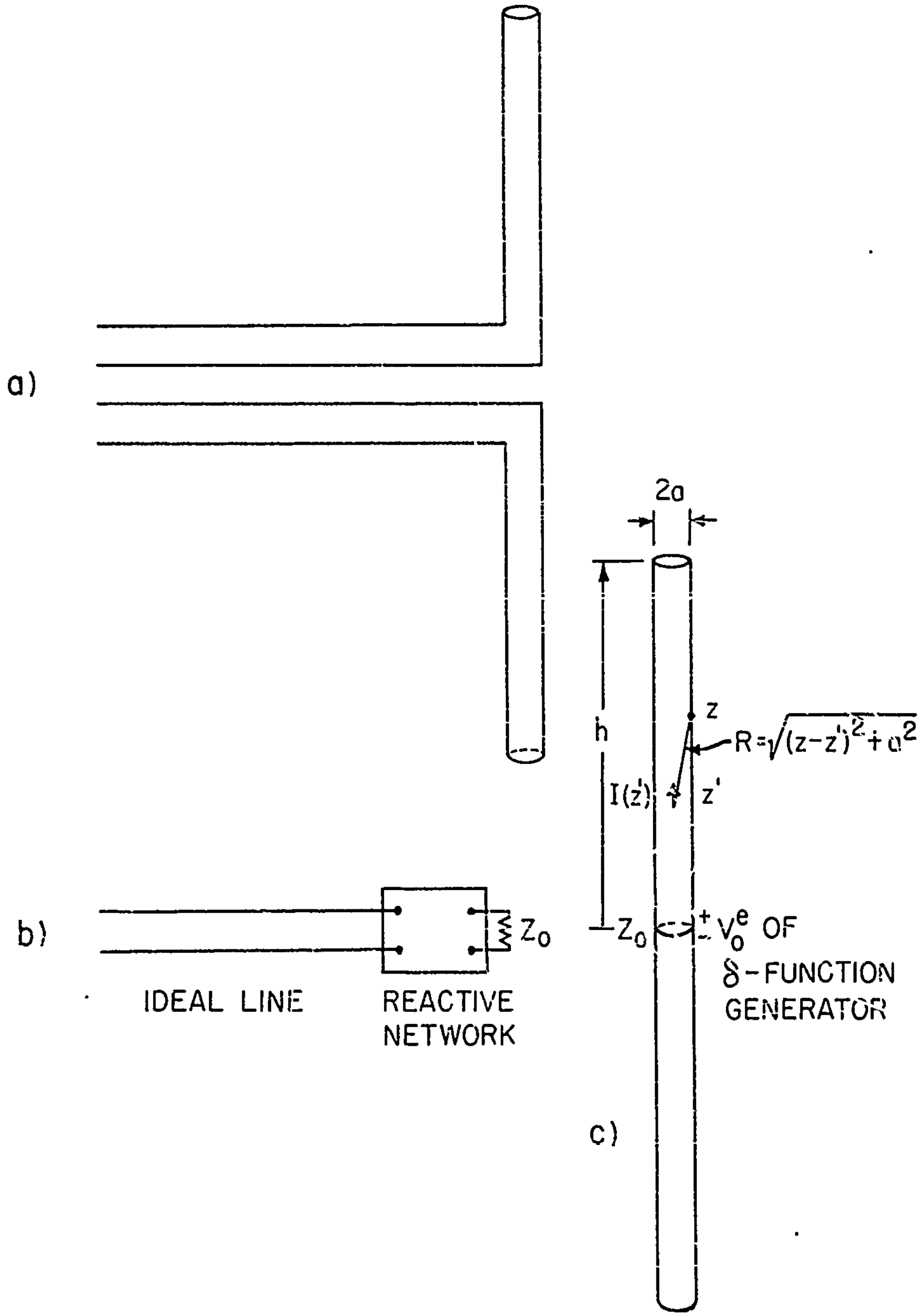

FIG. 1 a) ANTENNA DRIVEN FROM TWO-WIRE LINE
b) APPROXIMATELY EQUIVALENT LINE
c) APPROXIMATELY ANTENNA

treated as an impressed field that is concentrated in a ring around the center of the antenna, in the form $E_z^e(z)\,\delta(z)$, where $\delta(z)$ is the Dirac delta function. Clearly, this emf is equivalent to a discontinuity in scalar potential across a pair of knife edges separated by a vanishingly small distance at $z = 0$. Such an idealized generator at the center of the antenna evidently includes an infinite capacitance across the knife edges, so that the input susceptance must also be infinite. It was shown by Wu and King [15] that in principle the infinite current associated with this capacitance may be subtracted out. Moreover, since it is confined to an extremely short distance adjacent to the knife edges, it is in practice automatically omitted from the total current when this is determined approximately by any method of solution that represents the current by a few terms in a series of continuous functions. Thus, the practical problem may be approximated by an isolated cylindrical antenna with a delta-function generator at its center, as shown in Fig. 1c. The impedance of this antenna, after the knife-edge current has been subtracted out or omitted, is the lumped load for a transmission line with a suitable terminal-zone network. The nature of the lumped, corrective networks required for different connections to various types of lines is discussed elsewhere [16].

The serious attempt to determine the distribution of current in a thin cylindrical antenna by analytical means rather than by assuming it empirically, as is still commonly done in the so-called emf method, begins with the work of L. V. King [17] and especially of Hallén [18] whose integral equation is the basis of most modern theories. For a perfectly conducting tube of very small wall thickness and radius a that extends from $z = -h$ to $z = h$, the integral equation may be expressed in the form

$$4\pi\mu_0^{-1} A_z(z) = \int_{-h}^{h} I(z')\, K(z,\, z')\, dz'$$

$$= \frac{-j4\pi}{\zeta_0}\left[C\cos k_0 z + \tfrac{1}{2} V_0^e \sin k_0 |z|\right] \qquad (1)$$

where, as shown in Fig. 1c, $I(z')$ is the total axial current at z' .

If $I(z)$ includes currents on the inner and outer surfaces of the tube, it follows that

$$I(h) = 0 \tag{2}$$

V_0^e at the center of the antenna is that of an idealized delta-function generator. $A_z(z)$ is the vector potential at the surface of the cylinder. It is in the Lorentz gauge that satisfies the Sommerfeld radiation condition. The kernel K is given by

$$K(z,z') = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{e^{-jk_0R}}{R}\,d\theta' \doteq \frac{e^{-jk_0R_1}}{R_1} \tag{3}$$

$$\text{where} \quad R = \sqrt{(z-z')^2 + (2a\sin\tfrac{\theta'}{2})^2} \quad , \quad R_1 = \sqrt{(z-z')^2 + a^2} \quad . \tag{4}$$

As usual, μ_0 is the permeability; $h_0 \doteq 120\pi$ ohms, ζ is the characteristic impedance of free space; $k_0 = \omega/c$ is the wave number C is a constant to be determined from the boundary condition (2).

Approximate solutions of integral equations substantially like (1) have been obtained by Hallén and others by a method of iteration which depends implicitly on the observation that the ratio $A_z(z)/I(z)$ of the vector potential to the current along the antenna is approximately constant and predominantly real. Solutions of this type appear in the form

$$I(z) = \frac{j2\pi V_0^e}{\zeta_0 \Psi}\left[\frac{M_0(z) + M_1(z)/\Psi + M_2(z)/\Psi^2 + \cdots}{F_0(h) + F_1(h)/\Psi + F_2(h)/\Psi^2 + \cdots}\right] \tag{5}$$

where

$$M_0(z) = \sin k_0(h - |z|) \ , \quad F_0(h) = \cos k_0 h \ . \tag{6}$$

The first-order terms, $M_1(z)$ and $F_1(h)$ may be expressed in terms of generalized sine and cosine integrals, the higher order terms are more complicated and must be evaluated by numerical methods. Unfortunately, even with the most sophisticated definition of the expansion parameter Ψ, at least a second-order solution is required in order to obtain quantitative accuracy

for antennas with electrical lengths in the range $0 \leq k_o h \leq 2\pi$. A solution of this type is that of King and Middleton [8, 19] in which Ψ is defined to be the absolute value of the function

$$\Psi(z) = M_o^{-1}(z) \int_{-h}^{h} M_o(z')K(z,z')dz' \tag{6}$$

at the point where $M_o(z)$ has its greatest value in the range $0 \leq z \leq h$. Extensive computations of the impedances [19] of cylindrical antennas in the range of electrical half-lengths given by $0 \leq k_o h \leq 7$ show the second-order King-Middleton values to be in good agreement with experiment. The distributions of current for selected lengths have also been computed, but the second-order formula is far too complicated for the convenient evaluation of electromagnetic fields. For this purpose the rather crude zero-order term has been used.

The quantitative accuracy of second-order results calculated from (5) in the King-Middleton ~~from~~ form have been verified theoretically on the one hand by the comparable variational solutions of the integral equation (1) by Storer [20] and Tai [21]; and, on the other hand, by the detailed study of the integral equation with Fourier series methods by Duncan and Hinchey [22]. These investigators converted the integral equation into a set of simultaneous equations with the Fourier coefficients of the current distribution as the unknowns and then carried out calculations to the 25th-order. The numerical results for $k_o h = \pi/2$ and π and with $h/a = 60$ and 500π differ by only about 2 percent or less from the King-Middleton values. Significantly, even with a solution of such high order, there is still no sign of the large current associated with the knife edges at the driving point of the delta-function generator.

The reason for the rather slow convergence of the series in (5) even with a rather carefully selected expansion parameter has been traced by Wu [23] to the method of evaluation of the arbitrary constant C in terms of the boundary condition (2). Unfortunately, it is precisely at $z = h$ where the ratio of vector potential to current departs most from the assumed constant value--it actually becomes infinite at this point. It is the zero-order form of C, namely, $C = -\frac{V_o^e}{2} \tan k_o h$, which leads to the zero-order distribution, $\sin k_o(h - |z|)$.

It turns out that, for the input current and the current along most of the antenna, a better value of C would be: $C = -\frac{V_0^e}{2} \tan k_0(h + \delta)$ where δ is a small length that increases with the radius of the antenna. Evidently, such a value of C leads to a small non-vanishing zero-order current at $z = h$ in violation of (2). Primarily owing to the rather poor approximation in the zero-order value of C, the solution (5) even in second-order does not maintain its accuracy as the length of the antenna is increased. In order to handle specifically the very long antenna, Wu [23] developed an asymptotic solution of the integral equation (1) based on the Wiener-Hopf technique and a method of evaluating C which properly locates the distribution of current along the antenna, instead of requiring it to vanish at the end. The impedance of long antennas has been computed from the new formula by Beaton and Wu [24] for electrical lengths up to $k_0h = 30$. The new theory does not yield a simple expression for the current for use in the evaluation of electromagnetic fields. However, the radiation field can be obtained directly from the Fourier transform of the current.

The major characteristics of the radiation field of dipoles in air may be determined with reasonable accuracy from the simple sinusoidal distribution that is the leading term in the iterated solutions for the current. However, this zero-order current has the serious defect that its value at the driving point is at best a rough approximation of the correct input current. Moreover, if it is used to calculate the radiated power with the Poynting-vector theorem, the result may be in error by as much as 50 percent, if it is assumed to apply to center-driven antennas with practically significant radii. It follows that even if the field patterns calculated from a sinusoidally distributed current are an acceptable approximation, the power apparently supplied to the antenna at its terminals and the power radiated may be grossly in error and mutually inconsistent. If the input admittance is known accurately from a higher-order theory, these difficulties are not serious for antennas in air, since the correct total power supplied is then available and it is known without further calculation that this is equal to the power radiated. When an antenna is immersed in a conducting medium the problem is much more complicated, since power is dissipated throughout the medium.

What is required for the quantitatively accurate representation of the circuit and field properties of dipole antennas and arrays is a formula for the current that is sufficiently simple to permit the evaluation of the field and at the same time sufficiently accurate to yield good approximations of the input admittance and of the radiated power. Such a formula has been derived by Storer [10] by variational methods and by King [14] with a modified iterative procedure. Since the former is not conveniently applied to more than one antenna, and a future study of coupled antennas in dissipative media is contemplated, attention is focussed on the latter, which is as useful for parallel arrays as for a single antenna.

The required relatively simple formula for the current is obtained as the approximate solution of a rearranged form of the integral equation (1), viz.,

$$4\pi\mu_o^{-1}[A_z(z) - A_z(h)] = \int_{-h}^{h} I(z')K_d(z,z')dz'$$

$$= \frac{j4\pi}{\zeta_o F_o(h)}\left[U F_{oz} + \tfrac{1}{2}V_o^e M_{oz}\right] \qquad (7)$$

where the new kernel is

$$K_d(z,z') = K(z,z') - K(h,z') \qquad (8)$$

and

$$U = \frac{-j\omega}{k_o} A_z(h) = \frac{-j\zeta_o}{4\pi}\int_{-h}^{h} I(z')\,K(h,z')\,dz \;. \qquad (9)$$

The shorthand symbols

$$F_{oz} = \cos k_o z - \cos k_o h \cdot \qquad (10)$$

and

$$M_{oz} = \sin k_o (h - |z|) \qquad (11)$$

are used, together with $F_o(h)$ which is defined in (6). The advantages of the rearranged form (7) of the integral equation (1) are several. The integral is, as indicated, proportional to the vector potential difference rather than to the

vector potential itself, so that it vanishes at $z = h$ as does the current. The right-hand member of the equation is the sum of two terms that are individually related to well-known distributions of current and vector potential difference. The shifted cosine, F_{oz}, is a close approximation of the current and the vector potential difference along an unloaded receiving antenna in the plane wave front of a distant transmitting antenna. The sine term, M_{oz}, is the zero-order distribution along a center-driven antenna; it is much more exactly the distribution of both current and vector potential along an ideal two-wire transmission line with an open end at $z = h$ and a delta-function generator at $z = 0$. These facts suggest that F_{oz} may be interpreted as an approximation of the distribution of current or of the vector potential difference that is maintained by the interaction of the more widely separated elements in an antenna, whereas M_{oz} is the distribution maintained by a generator when, as in the transmission line, there is no significant interaction between widely separated current elements. This interpretation is confirmed by the fact that in (7) M_{oz} has the amplitude coefficient V_o^e, the actual driving voltage of the generator, whereas the coefficient U of F_{oz} is proportional to that part of the vector potential that has been subtracted out on the left because it is active along the entire antenna. In general and as a first approximation, a concentrated generator excites a current with the distribution M_{oz}; a distributed field excites a current with the distribution F_{oz}.

An approximate solution of (7) may be obtained if the integral is separated into two parts of which the one varies like M_{oz}, the other like F_{oz}. This separation is easily accomplished by inspection, once the kernel has been expanded as follows:

$$K_d(z, z') = K_{dR}(z, z') + jK_{dI}(z, z') \tag{12a}$$

where

$$K_{dR}(z, z') = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left[\frac{\cos k_o R}{R} - \frac{\cos k_o R_h}{R_h} \right] d\theta' \doteq \frac{\cos k_o R_1}{R_1} - \frac{\cos k_o R_{1h}}{R_{1h}} \tag{12b}$$

and

$$K_{dI}(z,z') = -\frac{1}{2\pi}\int_{-\pi}^{\pi}\left[\frac{\sin k_oR}{R} - \frac{\sin k_oR_h}{R_h}\right]d\theta' \doteq -\left[\frac{\sin k_oR_1}{R_1} - \frac{\sin k_oR_{1h}}{R_{1h}}\right]. \tag{12c}$$

The subscript h on R and R_1 denotes the values defined in (4) with $z = h$. Since R and R_1 become very small and $K_{dR}(z,z')$ correspondingly very large when z' is near z, it follows that the principal contributions to the part of the integral that has $K_{dR}(z,z')$ as kernel come from elements of current very near $z' = z$. This means that the part $A_{zR}(z)$ of $A_z(z)$ that depends on $K_{dR}(z,z')$ varies like $I(z)$. On the other hand, since $K_{dI}(z,z')$ is very small when z' is near z, the principal contributions to that part $A_{zI}(z)$ of $A_z(z)$ that depends on $K_{dI}(z,z')$ come from all the elements of current that are at some distance from z. If it is now assumed that the current is the sum of two parts,

$$I(z) = I_U(z) + I_V(z) \tag{13}$$

which by definition have the leading terms

$$I_U(z) \sim F_{oz}\ ; \qquad I_V(z) \sim M_{oz} \tag{14}$$

it is clear that (7) may be separated reasonably into the following parts:

$$\int_{-h}^{h}[I_U(z')K_d(z,z') + jI_V(z')K_{dI}(z,z')]\,dz' \doteq \frac{j4\pi U}{\zeta_o F_o(h)}F_{oz} \tag{15}$$

$$\int_{-h}^{h} I_V(z')K_{dR}(z,z')\,dz' \doteq \frac{j2\pi V_o^e}{\zeta_o F_o(h)}M_{oz} \tag{16}$$

Each of these integral equations may now be solved by iteration in the King-Middleton manner, and their solutions for $I_U(z)$ and $I_V(z)$ combined in (18) to give $I(z)$. The formula for the current so obtained may then be substituted in (9) in order to evaluate the constant U in terms of V_o^e. The result is

$$I(z) = \frac{j2\pi V_o^e}{\zeta_o \Psi_{dR}}\left[\frac{\sin k_o(h-|z|) + T(h)(\cos k_oz - \cos k_oh)}{\cos k_oh}\right] + \text{higher order terms} \tag{17}$$

The indeterminate form obtained when $k_o h = \pi/2$ may be evaluated to give

$$I(z) = \frac{j2\pi V_o^e}{\zeta_o \Psi_{dR}} \left[\sin k_o |z| - 1 + T'\left(\frac{\lambda}{4}\right) \cos k_o z \right] + \text{higher order terms} \tag{18}$$

The real expansion parameter Ψ_{dR} and the complex constant $T(h)$ are expressed in terms of tabulated sine, cosine, and exponential integrals. They are functions of h/a and $k_o h$. Explicit formulas and numerical values are in the literature [14, 25]. The higher order terms in (17) and (18) involve essentially the same integrals as the corresponding terms in the King-Middleton solution (5), but their contributions to $I(z)$, at least in the first- and second-order terms, are very much smaller. It follows that the quasi-zero-order terms in (17) and (18) have an accuracy that lies somewhere between the first- and the second-order solution in the form (5) in the range $0 \leq k_o h < 3\pi/2$. Moreover, the error is largely in the susceptive part of the current in a small region very near the driving point. As a specific example, consider a half-wave antenna with $h/a = 75$. The distribution of current as obtained from (18) is shown in Fig. 2. The corresponding admittance and impedance are

$$[Y_o]_o = (9.87 - j4.67) \times 10^{-3} \text{mhos} ; [Z_o]_o = 82.8 + j39.2 \,\text{ohms} \quad .$$

The second-order theoretical values (which are in good agreement with experiment) are:

$$[Y_o]_2 = (9.38 - j4.52) \times 10^{-3} \text{mhos} ; [Z_o]_2 = 86.5 + j41.7 \,\text{ohms} \quad .$$

The values obtained by the emf method independent of h/a are

$$Y_{emf} = (10.22 - j5.94) \times 10^{-3} \,\text{mhos}; Z_{emf} = 73.13 + j42.5 \,\text{ohms} \quad .$$

As compared with the second-order value, the admittance given by (18) for the half-wave dipole is about 5 percent in error in the conductance, 3 percent in the susceptance; the corresponding errors in the values obtained by the emf method are 9 percent and 31 percent. For greater lengths the results of the emf method deteriorate very rapidly.

It may be concluded that (17) and (18) combine reasonable accuracy with simplicity in both the distribution of current and the admittance.

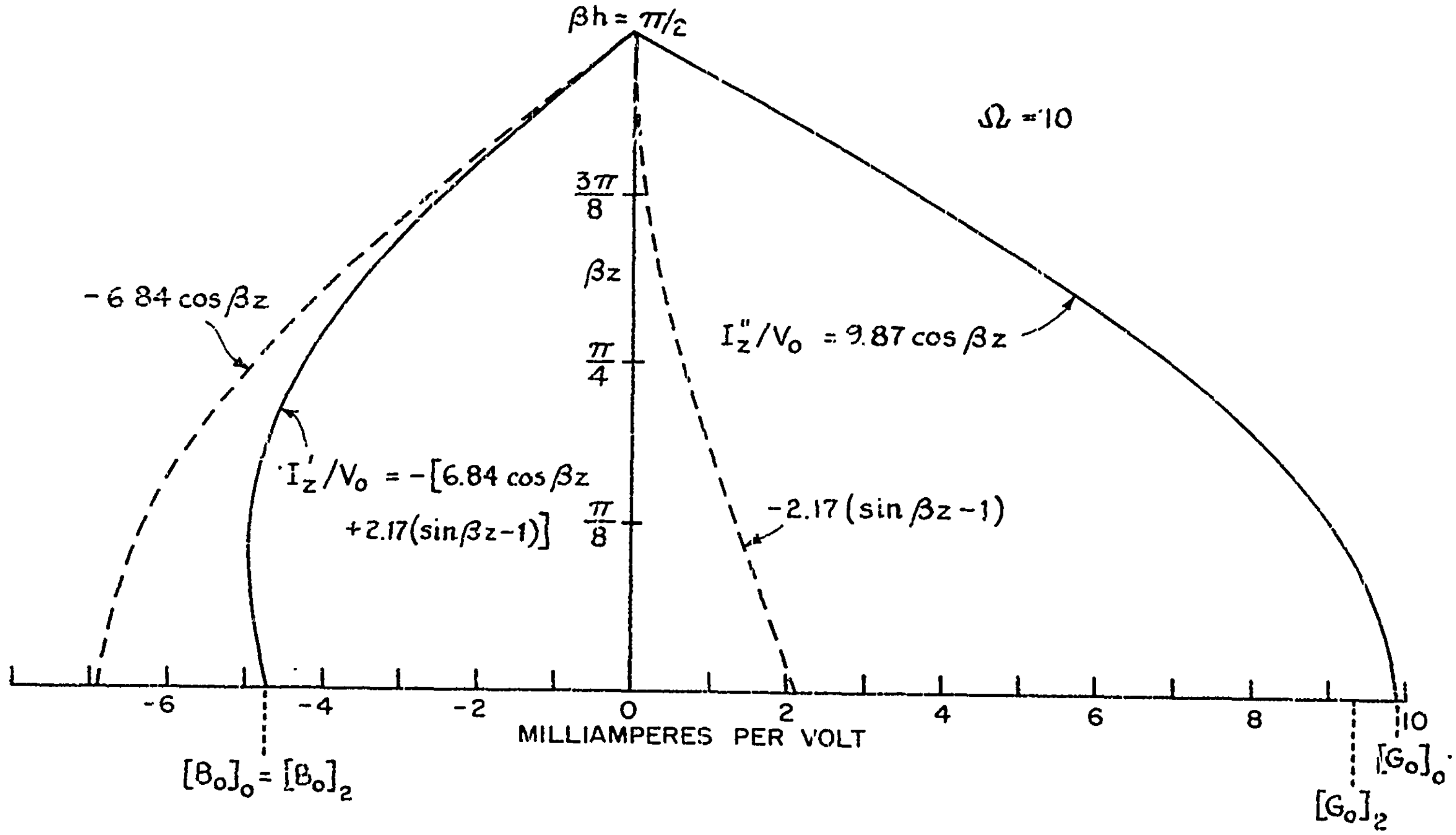

FIG. 2 QUASI-ZEROTH-ORDER CURRENT

The Cylindrical Antenna in a Dissipative Medium

1. The General Case

The physical problem under consideration is to determine the current in a highly conducting cylindrical antenna immersed in a homogeneous, isotropic, dissipative medium of great extent. The antenna is center-driven, for example, from a shielded-pair line that lies in the neutral plane. As with the antenna in air, it is convenient to approximate this configuration by an ideal line with the impedance of the antenna and a suitable terminal-zone network as a lumped load. For this purpose the antenna is imagined center-driven by a delta-function generator. In this case, the knife edges of the generator terminals are equivalent to an infinite admittance that includes both a capacitance and a conductance. The infinite current that is associated with the charging of the knife-edge capacitance and that crosses from one edge to the other by way of the dissipative medium may again be subtracted out in principle, since it is confined to an extremely short distance on each side of the generator. In practice, it is excluded from a solution that approximates the current in the antenna by a series of continuous functions.

The integral equation for a perfectly conducting cylindrical antenna immersed in an infinite, homogeneous, isotropic, dissipative medium and center-driven by a delta-function generator is formally like the equation for the same antenna in air if the complex dielectric factor $\xi = \epsilon_e - j\frac{\sigma_e}{\omega}$ is substituted for ϵ_o and μ replaces μ_o. It is assumed that μ is real. This is equivalent to the replacement of the real wave number $k_o = \omega\sqrt{\mu_o\epsilon_o}$ by the complex propagation constant

$$k = \beta - j\alpha = \omega\sqrt{\mu\epsilon_e}\,\sqrt{1 - jp} = \omega\sqrt{\mu\epsilon_e}\,[f(p) - jg(p)] \tag{19}$$

where $p = \sigma_e/\omega\epsilon_e$ and $f(p) = \cosh(\frac{1}{2}\sinh^{-1}p)$, $g(p) = \sinh(\frac{1}{2}\sinh^{-1}p)$.

Tables of the functions $f(p)$ and $g(p)$ are available in the literature [26]. In addition, the real characteristic impedance $\zeta_o = \sqrt{\mu_o/\epsilon_o} = \omega\mu_o/k_o$ is replaced by the complex value

$$\zeta = \sqrt{\frac{\mu}{\xi}} = \frac{\omega\mu}{k} = \zeta_a e^{j\phi} \tag{20a}$$

where

$$\zeta_a = \frac{\omega\mu}{\sqrt{\beta^2 + a^2}} \, , \quad \phi = \tan^{-1}\frac{a}{\beta} \quad . \tag{20b}$$

In these relations ϵ_e and σ_e are the real effective permittivity and conductivity, respectively. In terms of the complex permittivity $\epsilon = \epsilon' - j\epsilon''$ and the complex conductivity $\sigma = \sigma' - j\sigma''$ the real, effective values are given by

$$\epsilon_e = \epsilon_o \epsilon_{er} = \epsilon' - \frac{\sigma''}{\omega} \, , \qquad \sigma_e = \sigma' + \omega\epsilon'' \tag{21}$$

In the rearranged form (7) the integral equation is

$$\int_{-h}^{h} I(z')\, K_{kd}(z, z')\, dz' = \frac{j4\pi\omega\mu}{kF_k(h)} \left[U_k F_{kz} + \frac{1}{2} V_o^e M_{kz} \right] \tag{22}$$

where

$$U_k = \frac{-j\omega\mu}{4\pi k} \int_{-h}^{h} I(z') K_k(h, z')\, dz' \tag{23}$$

$$F_k(h) = \cos kh = \cos\beta h \cosh a h + j \sin\beta h \sinh a h \tag{24}$$

$$F_{kz} = \cos kz - \cos kh$$

$$= (\cos\beta z \cosh a z - \cos\beta h \cosh a h) + j(\sin\beta z \sinh a z - \sin\beta h \sinh a h) \tag{25}$$

$$M_{kz} = \sin k(h - |z|)$$

$$= \sin\beta(h - |z|) \cosh a(h - |z|) - j \cos\beta(h - |z|) \sinh a(h - |z|) \tag{26}$$

The kernels are given by

$$K_{kd}(z, z') = K_k(z, z') - K_k(h, z') \doteq \frac{e^{-jkR_1}}{R_1} - \frac{e^{-jkR_{1h}}}{R_{1h}} \tag{27}$$

The difference kernel may be separated into two parts as follows:

$$K_{kR}(z, z') = K_{kR}(z, z') + jK_{kI}(z, z') \tag{28a}$$

where

$$K_{kR}(z,z') = \frac{\cos\beta R_1 \cosh\alpha R_1}{R_1} - \frac{\cos\beta R_h \cosh\alpha R_h}{R_h} \tag{28b}$$

$$K_{kI}(z,z') = -R_1^{-1}\left[\sin\beta R_1(\cosh\alpha R_1 - \sinh\alpha R_1) + j\cos\beta R_1 \sinh\alpha R_1\right]$$

$$+ R_{1h}^{-1}\left[\sin\beta R_{1h}(\cosh\alpha R_{1h} - \sinh\alpha R_{1h}) + j\cos\beta R_{1h}\sinh\alpha R_{1h}\right] \tag{28c}$$

Note that $K_{kI}(z,z')$ is not real when $\alpha \neq 0$; it does reduce to the real $K_{dI}(z,z')$ when $\alpha = 0$. This equation (22) has a much more complicated kernel than (7) and the right member is also more involved, since the distribution terms F_{kz} and M_{kz} are complex, and contain the additional parameter α.

The type of solution desired for the current in an antenna in a dissipative medium is one corresponding to (17) for the antenna in air that provides good approximations of both the distribution of current and the admittance in a form that is sufficiently simple to permit the direct integration of the integrals for the electromagnetic field. It is in order to obtain such a solution that the form (22) of the integral equation was chosen. The approximate solution of this equation may be carried out in a manner closely paralleling the procedure described for the antenna in air. Indeed, the kernel (27) has already been separated into two parts such that the one $K_{kR}(z,z')$, is very large when z' is near z so that the principal contributions to $A_{zR}(z)$ are from the currents near z; and the other, $K_{kI}(z,z')$, is relatively small near $z' = z$ so that the principal contributions to $A_{zI}(z)$ are from currents at some distance from z. As before in (13), the current may be expressed as the sum of two terms and the integral equation appropriately separated into two parts as in (15) and (16):

$$\int_{-h}^{h}\left[I_U(z')K_{kd}(z,z') + jI_V(z')K_{kI}(z,z')\right]dz' \doteq \frac{j4\pi k U_k}{\omega\mu F_k(h)} F_{kz} \tag{29}$$

$$\int_{-h}^{h} I_V(z')K_{kR}(z,z')\,dz' \doteq \frac{j2\pi k V_0^c}{\omega\mu F_k(h)} M_{kz} \quad . \tag{30}$$

These equations may be solved separately by iteration and the resulting solutions added to obtain $I(z)$ in terms of V_o^e and U_k. U_k may then be expressed in terms of V_o^e with the substitution of $I(z)$ in (24). The solution for the current is

$$I(z) = \frac{j2\pi k V_o^e}{\omega\mu\Psi_{kR}\cos kh}\left[\sin k(h - |z|) + T_k(h)(\cos kz - \cos kh)\right] + \begin{cases}\text{higher}\\ \text{order}\\ \text{terms}\end{cases} \qquad (31)$$

where Ψ_{kR} is a complex expansion parameter and $T_k(h)$ a complex constant that depends on h/a, βh, and α/β. Unfortunately, the integral functions that occur in $T_k(h)$ are not in general available in tabulated form. However, explicit evaluation has been carried out for the half-wave dipole [12] in a moderately conducting medium such that $\beta h = \pi/2$ and $\alpha h < 1$ and for the electrically short dipole [13] in an arbitrary medium defined by $\beta h \leq 0.3$, $\alpha \leq \beta$.

2. The Half-Wave Dipole

For the half-wave dipole, the approximations $\cosh \alpha h \doteq 1$ and $\sinh \alpha h \doteq \alpha h$ are made and (31) reduced to the form

$$\frac{I(z)}{V_o^e\sqrt{\epsilon_{er}}} = \frac{-j2\pi(1 - j\alpha/\beta)}{\zeta_o\Psi_{kR}}\left[\sin\beta|z| - 1 + T_k'\left(\frac{\lambda}{4}\right)\cos\beta z\right] + \text{higher order terms} \qquad (32)$$

where

$$T_k'\left(\frac{\lambda}{4}\right) = -\left[1 + T_k\left(\frac{\lambda}{4}\right)\right]\left[\frac{1 - j\alpha h}{j\alpha h}\right] \quad . \qquad (33)$$

The normalized current $I(z)/V_o^e\sqrt{\epsilon_{er}}$ is shown in Fig. 3 with $I(z) = I_z'' + jI_z'$ as a function of z with $2\alpha/\beta = \sigma_e/\omega\epsilon_o\epsilon_{er}$ as parameter. The normalized impedance $Z\sqrt{\epsilon_{er}}$ and admittance $Y/\sqrt{\epsilon_{er}}$ are shown in Fig. 4. Both the distributions of current and the impedance are in agreement with experimental results obtained by Iizuka. It may be added that measurements have been made with larger values of $\sigma_e/\omega\epsilon_o\epsilon_{er}$ than those used in the theoretical calculations represented in Fig. 4. These indicate that the resistance curve in Fig. 4 reaches a maximum as $\sigma_e/\omega\epsilon_o\epsilon_{er}$ is increased further and then bends down to approach zero. The corresponding distributions of current change from concave outward as in Fig. 3 to concave inward as $\sigma_e/\omega\epsilon_o\epsilon_{er}$ is increased.

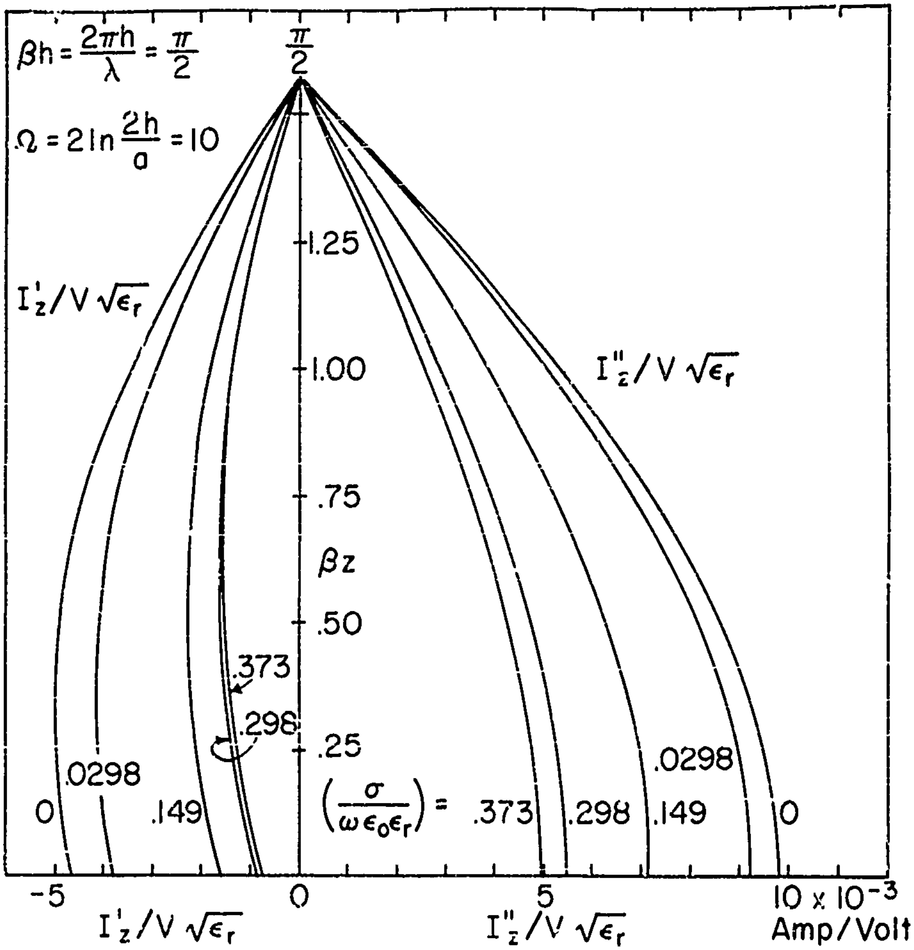

FIG. 3 NORMALIZED CURRENT $I_z = I''_z + j I'_z$ IN HALF-WAVE DIPOLE IN MEDIUM WITH CONDUCTIVITY σ AND DIELECTRIC CONSTANT $\epsilon = \epsilon_0 \epsilon_r$

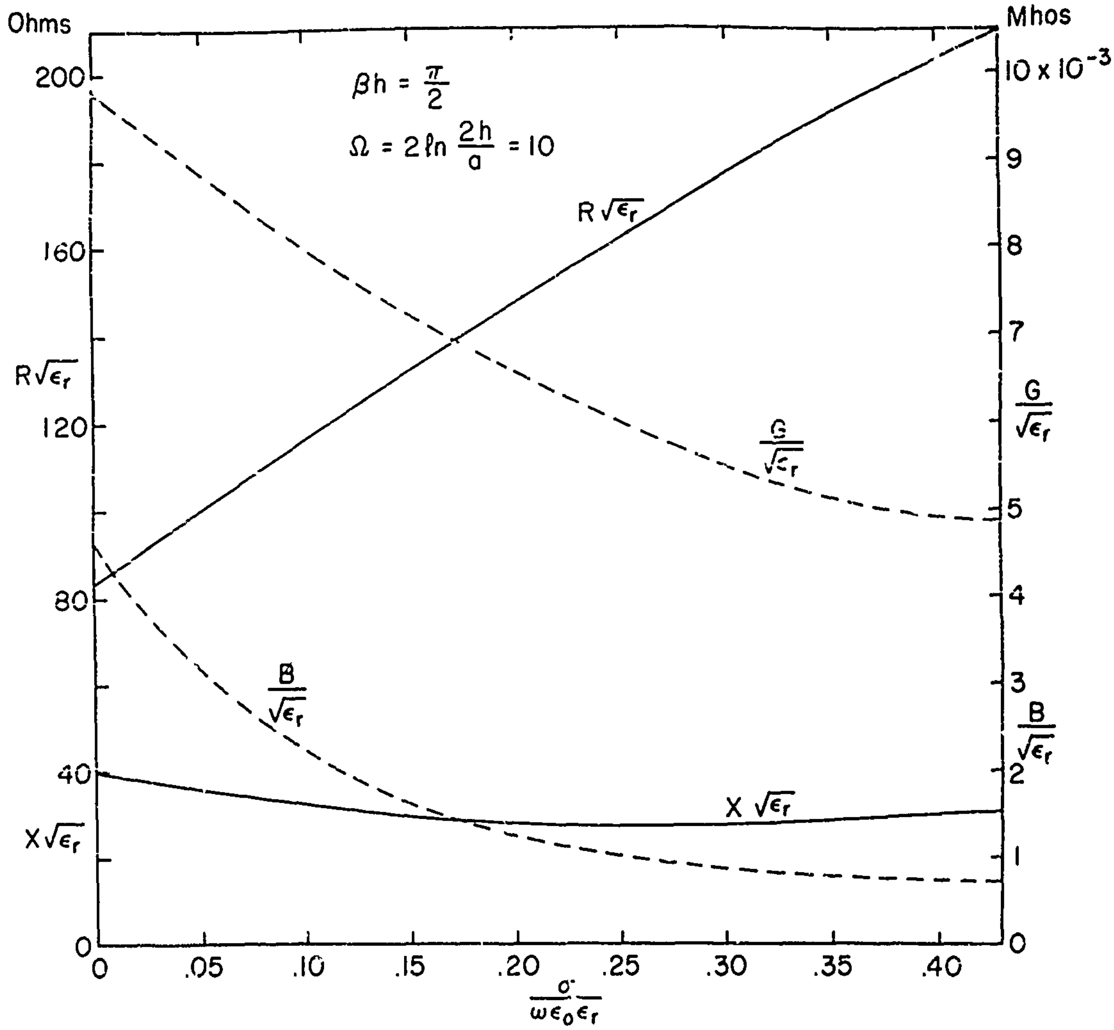

FIG. 4 IMPEDANCE AND ADMITTANCE OF A DIPOLE WITH ELECTRICAL HALF LENGTH $\beta h = \pi/2$ WHEN IMMERSED IN A MEDIUM WITH CONDUCTIVITY σ AND DIELECTRIC CONSTANT $\epsilon = \epsilon_0 \epsilon_r$; $\beta = \omega\sqrt{\mu\epsilon} = 2\pi/\lambda$

3. The Electrically Short Antenna

The current in the electrically short antenna defined by $\alpha h < 1$, $\beta h < 1$ is readily obtained from (31) as a series in powers of βh and αh. The first few terms are:

$$I(z) = \frac{j2\pi kh}{\zeta\Psi_{dR}}\left\{\left(1 + \frac{k^2h^2}{3}\right)\left(1 - \frac{|z|}{h}\right) + \frac{(3\ln 2 - 1)k^2h^2 - jk^3h^3}{3(\Omega - 3)}\left(1 - \frac{z^2}{h^2}\right)\right\} . \tag{34}$$

The corresponding admittance is

$$Y_o(k) = G_o(k) + jB_o(k) \tag{35a}$$

where

$$G_o(k) = \frac{2\pi}{\zeta_e\Psi_{dR}}\left\{\frac{2\alpha}{\beta}\left[\beta h + (2\beta^3h^3F/3)\left(1 - \frac{\alpha^2}{\beta^2}\right)\right] + \frac{\beta^4h^4}{3(\Omega - 3)}\left(1 - 10\frac{\alpha^2}{\beta^2} + 5\frac{\alpha^4}{\beta^4}\right)\right\} \tag{35b}$$

$$B_o(k) = \frac{2\pi}{\zeta_e\Psi_{dR}}\left\{\beta h\left(1 - \frac{\alpha^2}{\beta^2}\right) + \frac{\beta^3h^3F}{3}\left(1 - 6\frac{\alpha^2}{\beta^2} + \frac{\alpha^4}{\beta^4}\right) - \frac{\beta^4h^4}{3(\Omega - 3)}\frac{\alpha}{\beta}\left(5 - 10\frac{\alpha^2}{\beta^2} + \frac{\alpha^4}{\beta^4}\right)\right\} . \tag{35c}$$

In these formulas

$$F = 1 + \frac{3\ln 2 - 1}{\Omega - 3} = 1 + \frac{1.08}{\Omega - 3} \tag{36}$$

and

$$\Omega = 2\ln(2h/a) . \tag{37}$$

Also

$$\zeta = \frac{\omega\mu}{k} , \qquad \zeta_e = \frac{\omega\mu}{\beta} \tag{38}$$

These expressions are general and apply to all values of α and β that satisfy the relations $\beta h \leq 0.3$, $\alpha h \leq 0.3$. The fact that it is necessary to retain terms up to and including fourth powers of the small quantity βh is an indication of the complicated nature of the admittance of an antenna in a dissipative medium with no restriction on α. The reason is obvious: if $\alpha = 0$ as in a perfect dielectric, the leading terms are

$$G_o(k_o) \doteq \frac{2\pi\beta^4h^4}{3\zeta_e\Psi_{dR}(\Omega - 3)} \qquad B_o(k_o) \doteq \frac{2\pi\beta h}{\zeta_e\Psi_{dR}} . \tag{39}$$

On the other hand, when $\alpha = \beta$ as in salt water,

$$G_o(k) \doteq \frac{4\pi\beta h}{\zeta_e \Psi_{dR}} \qquad\qquad B_o(k) \doteq -\frac{4\pi F\beta^3 h^3}{3\zeta_e \Psi_{dR}} \tag{40}$$

where F is defined in (36). Note that $G_o(k_o)$ in (39) is a pure radiation conductance; it is very small compared with the susceptance. In (40) $G_o(k)$ is determined entirely by dissipation in the medium, the contribution from radiation is negligible; $G_o(k)$ is very large compared with $B_o(k)$. It is clear from the general expression (35a) that the radiation conductance predominates for only a very small range of α near zero. This indicates that the short dipole behaves as a radiating antenna only when α is very small; for larger values it is essentially a pair of electrodes.

The normalized impedance $\frac{Z_o(k)}{\zeta_e} = \frac{R_o(k)}{\zeta_e} + j\frac{X_o(k)}{\zeta_e}$ and admittance $Y_o(k)\zeta_e = G_o(k)\zeta_e + jB_o(k)\zeta_e$ of a short antenna with $\beta h = 0.3$ and $\Omega = 10$ are shown in Fig. 5 as a function of α/β. The curves for $R_o(k)/\zeta_e$ and $G_o(k)\zeta_e$ are also shown on a logarithmic scale in Fig. 6 in order to show their extremely rapid rise in the range of very small values of α/β. Note that $R_o(k)/\zeta_e$ has a maximum between $\alpha/\beta = 0.5$ and 0.6.

If the medium in which the admittance $Y_o(k)$ is measured is an ionized region such as the ionosphere, the conductance $G_o(k)$ and the susceptance $B_o(k)$ may be related directly to the concentration of electrons and to their collision frequency. Details are given elsewhere [13].

The Electromagnetic Field of an Antenna in a Dissipative Medium

1. The General Case and the Field at Distant Points

The electromagnetic field of a cylindrical antenna of half-length h with a total axial current I(z) when immersed in a dissipative medium may be determined from the vector potential

$$A_z(r) = \frac{\mu}{4\pi}\int_{-h}^{h} I(z')\,\frac{e^{-jkR}}{R}\,dz' \tag{41}$$

where

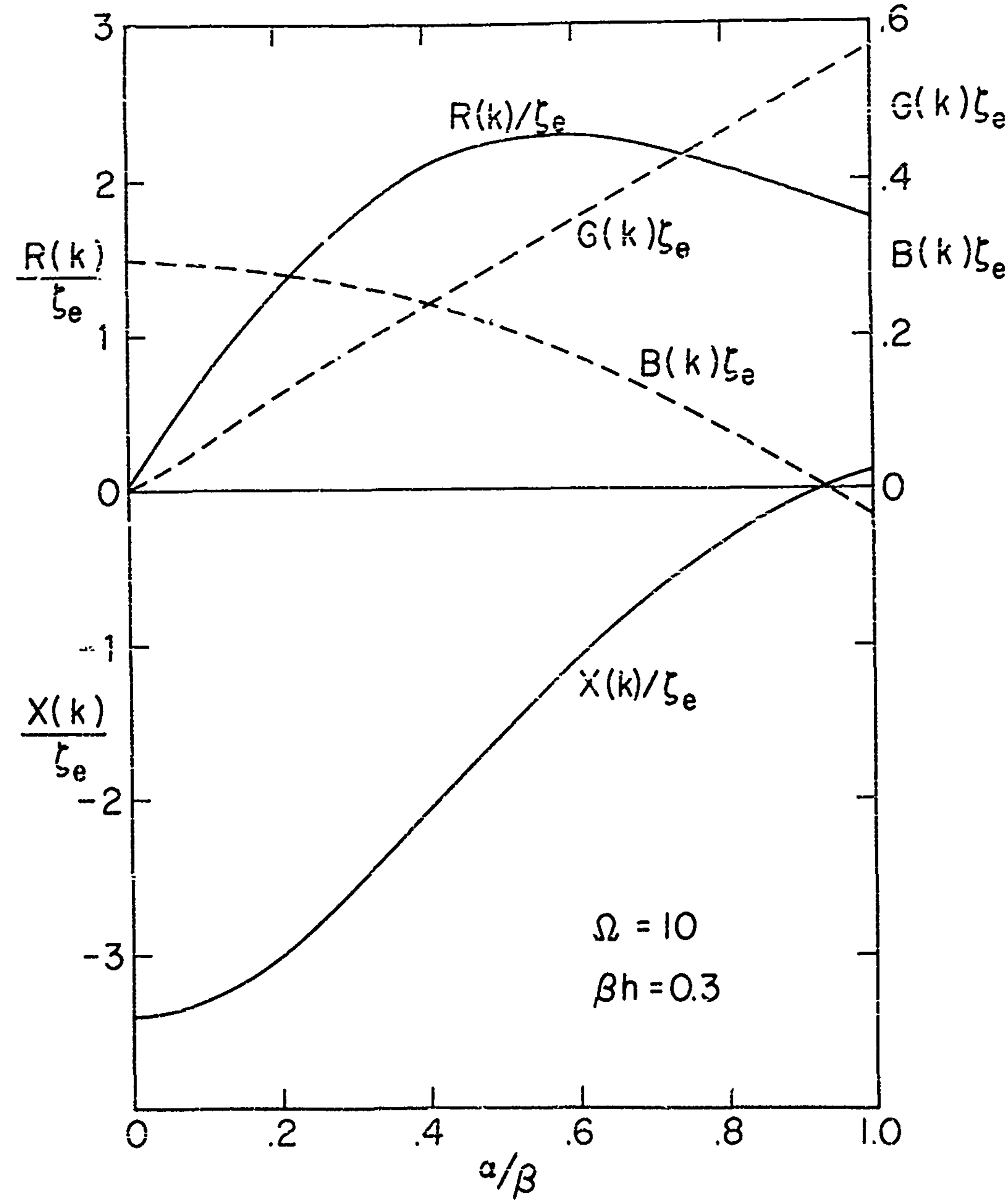

FIG. 5 NORMALIZED IMPEDANCE AND ADMITTANCE OF SHORT ANTENNA IN DISSIPATIVE MEDIUM

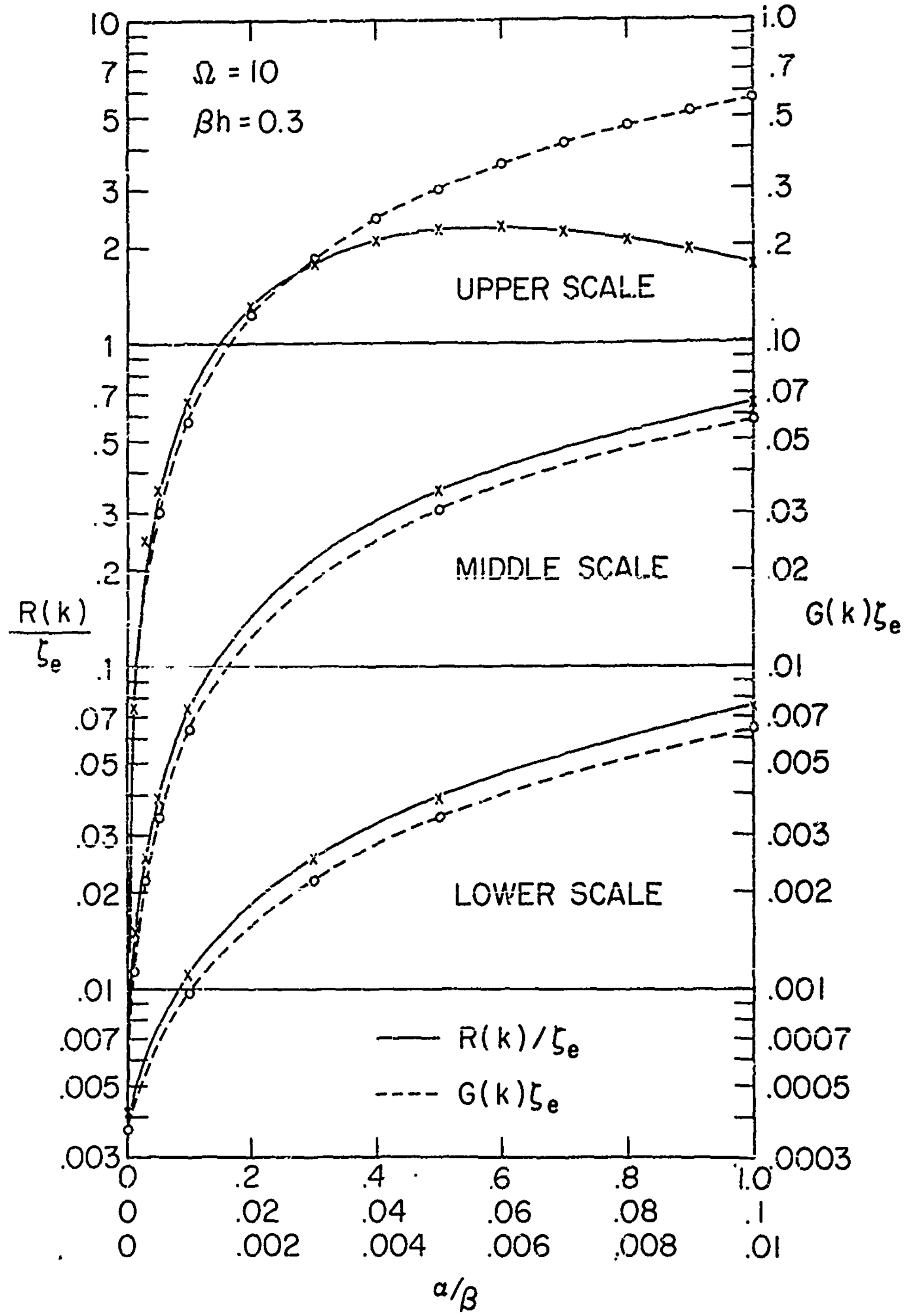

FIG. 6 NORMALIZED CONDUCTANCE AND RESISTANCE OF SHORT ANTENNA IN DISSIPATIVE MEDIUM

$$R = \sqrt{(z - z')^2 + \rho^2 + a^2 - 2a\cos\theta'}$$

$$\doteq \sqrt{(z - z')^2 + \rho^2} \tag{42}$$

when expressed in the cylindrical coordinates ρ, θ, z. The formula (41) may be used to determine the vector potential at any point except within distances of the ends of the antenna comparable with its radius a. At distances $r = \sqrt{z^2 + \rho^2}$ from the center of the antenna that are large compared with the half-length h, that is, when

$$r^2 \gg h^2, \quad R \doteq r - z'\cos\theta, \tag{43}$$

an approximate formula for the vector potential in the radiation zone may be used. This is

$$A_z^r(r) = \frac{\mu}{4\pi}\frac{e^{-jkr}}{r}\int_{-h}^{h} I(z')\,e^{jkz'\cos\theta}\,dz' \quad . \tag{44}$$

The corresponding non-vanishing cylindrical components of the magnetic and electric fields are given by

$$B_\theta(r) = -\frac{\partial A_z(r)}{\partial \rho} \;;\; E_\rho(r) = \frac{jk^2}{\omega}\frac{\partial^2 A_z(r)}{\partial\rho\,\partial_1 z} \;;\; E_z(r) = \frac{-jk^2}{\omega}\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial A_z(r)}{\partial\rho}\right). \tag{45}$$

In the radiation zone the significant components are conveniently expressed in the spherical coordinates, r, θ, Φ. They are given by

$$B_\Phi^r(r) = -jk\,A_z^r(r)\sin\theta \;; \qquad E_\theta^r(r) = -j\omega A_z^r(r)\sin\theta \quad . \tag{46}$$

In order to determine the complete electromagnetic field it is necessary to evaluate the integral in (41) or at least the derivatives of this integral required in (45). For the radiation field at distances that satisfy (43) it is sufficient to evaluate the simpler integral in (44). A simple and reasonably accurate representation of the current in a cylindrical antenna of zero to moderate length is that given in (31). Unfortunately, if this distribution is

substituted in (41) and (45) is evaluated, one of the three terms involved cannot be obtained in closed form. On the other hand, the integration of (44) is straightforward. With (46) the result is

$$\frac{\omega}{k} B^r(\vec{r}) = E_\theta^r(\vec{r}) = \frac{V_o^e e^{-jkr}}{\Psi_{kr} r \cos kh} [F_m(\theta, kh) + T_k(h) G_m(\theta, kh)] \tag{47}$$

where

$$F_m(\theta, kh) = \frac{\cos(kh \cos\theta) - \cos kh}{\sin\theta} \tag{48}$$

$$G_m(\theta, kh) = \frac{\sin kh \cos(kh\cos\theta)\cos\theta - \cos kh \sin(kh\cos\theta)}{\sin\theta \cos\theta} \quad . \tag{49}$$

These expressions can be separated into real and imaginary parts and the electric and magnetic fields evaluated. However, since the amplitude decreases as e^{-ar}/r, it is evident that the range of r in which the field is significant and in which (43) is satisfied is not great unless a is sufficiently small to satisfy the conditions

$$a^2h^2 << 1 \ , \quad \sinh ah \doteq ah \ , \quad \cosh ah \doteq 1 \quad . \tag{50}$$

Nevertheless, the general directional properties of the field not very close to the antenna may be determined from (48) and (49) even when $a = \beta$. When (50) is satisfied the field factors defined in (48) and (49) become somewhat simpler. Specifically,

$$F_m(\theta, kh) = F_{mr}(\theta, kh) + jahF_{mi}(\theta, kh) \tag{51a}$$

where

$$F_{mr}(\theta, kh) = \frac{\cos(\beta h \cos\theta) - \cos\beta h}{\sin\theta} \tag{51b}$$

$$F_{mi}(\theta, kh) = \frac{\sin(\beta h\cos\theta)\cos\theta - \sin\beta h}{\sin\theta} \quad .$$

Similarly,

$$G_m(\theta, kh) = G_{mr}(\theta, kh) + jah\, F_{mi}(\theta, kh) \tag{52a}$$

where

$$G_{mr}(\theta, kh) = \frac{\sin\beta h \cos(\beta h \cos\theta)\cos\theta - \cos\beta h \sin(\beta h \cos\theta)}{\sin\theta\cos\theta} \tag{52b}$$

$$G_{mi}(\theta, kh) = \sin\beta h \sin(\beta h \cos\theta)\tan\theta \quad . \tag{52c}$$

Also in (47),

$$\cos kh = \cos\beta h + ja h \sin\beta h \quad . \tag{53}$$

When $\beta h = \pi/2$, (51) and (52) become

$$F_m(\theta, \tfrac{1}{2}\pi - jah) = F_m(\theta, \tfrac{\pi}{2}) + jah\left[\frac{\sin(\frac{\pi}{2}\cos\theta)\cos\theta - 1}{\sin\theta}\right] \tag{54a}$$

$$G_m(\theta, \tfrac{1}{2}\pi - jah) = F_m(\theta, \tfrac{\pi}{2}) + jah \sin(\tfrac{\pi}{2}\cos\theta)\tan\theta \tag{54b}$$

where

$$F_m(\theta, \tfrac{\pi}{2}) = \frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta} \tag{54c}$$

Since from (53) $\cos kh \doteq jah$, it follows with the readily verified approximation (that is valid only when $\beta h = \pi/2$)

$$\tan\theta \sin(\tfrac{\pi}{2}\cos\theta) \doteq \tfrac{\pi}{2}F_m(\theta, \tfrac{\pi}{2}) \tag{55}$$

and the definition (33), that (47) becomes

$$E^r_\theta \doteq -\frac{V^e_o e^{-(\alpha + j\beta)r}}{\Psi_{kr}\, r}\left[H_m(\theta, \tfrac{\pi}{2}) + T'_k(\tfrac{\lambda}{4})\left(\frac{1 - jah\pi/2}{1 - jah}\right)F_m(\theta, \tfrac{\pi}{2})\right] \tag{56a}$$

where

$$H_m(\theta, \tfrac{\pi}{2}) = \frac{\cos\theta - \sin(\frac{\pi}{2}\cos\theta)}{\sin\theta\cos\theta} \doteq (1 - \tfrac{\pi}{2})\, F_m(\theta, \tfrac{\pi}{2}) \quad . \tag{56b}$$

This is the complete field generated by the current (32). Note that $H_m(\theta, \frac{\pi}{2})$ is the part of the field contributed by the component of current $\sin\beta|z| - 1$. The factor $(1 - jah\pi/2)/(1 - jah)$ in (57) as compared with (32) is a consequence of the attenuation of the field over the small distances $z'\cos\theta$ in

$R = r - z' \cos\theta$. The field factors $F_m(\theta, \frac{\pi}{2})$ and $H_m(\theta, \frac{\pi}{2})$ are shown in Fig. 7.

If the approximate relation on the right in (56b) is substituted in (56a), the approximate electric field is given by

$$E_\theta^r = -\frac{V_o^e e^{-(\alpha + j\beta)r}}{\Psi_{kr} r}\left[1 - \frac{\pi}{2} + T_k'(\frac{\lambda}{4})\left(\frac{1 - j\alpha h\pi/2}{1 - j\alpha h}\right)\right] F_m(\theta, \frac{\pi}{2}) \ . \tag{57}$$

This field would be maintained accurately by the current

$$\frac{I(z)}{V_o^e\sqrt{\epsilon_{er}}} = \frac{-j2\pi(1 - j\alpha/\beta)}{\zeta_o \Psi_{kr}}\left[1 - \frac{\pi}{2} + T_k'(\frac{\lambda}{4})\right] \cos\beta z \ , \tag{58}$$

and only approximately by (32). Note that in (32) the input current is proportional to $[-1 + T_k'(\frac{\lambda}{4})]$ whereas in (58) it is proportional to $[1 - \frac{\pi}{2} + T_k'(\frac{\lambda}{4})]$. Since the former gives the correct value, the latter is evidently in error. The explanation for this discrepancy is clear. The correct current in (32) yields the correct field in (56a) and the correct radiated power. If the current is approximated by a purely cosinusoidal distribution, its amplitude can be adjusted so that the <u>input current</u> is correct with the factor $[-1 + T_k'(\frac{\lambda}{4})]$, or so that the <u>field</u> and the <u>power radiated</u> are correct with the factor $[1 - \frac{\pi}{2} + T_k'(\frac{\lambda}{4})]$. In the former case the power radiated is too small, in the latter case the input current is too large.

Graphs of the electric field defined in (47) when $\beta h = \pi$, $\alpha = 0$, $h/a = 75$ are shown in Fig. 8. It may be concluded from the forms of (54a) and (54b) that the field patterns when αh is small do not differ greatly from those when $\alpha = 0$. Significant differences may be anticipated when αh is not small compared with one, in particular, when α is of the same order of magnitude as β . For their evaluation the complex parameter $T_k(h)$ must be computed.

2. The Field Near the Dipole; Approximate Currents and Cylindrical Coordinates

In order to determine the electromagnetic field nearer the antenna than is permitted by the condition (43) it is necessary to carry out extensive numerical computations for individual cases or to make approximations in the distribution of current in order to obtain an integrable form. Fortunately, the latter

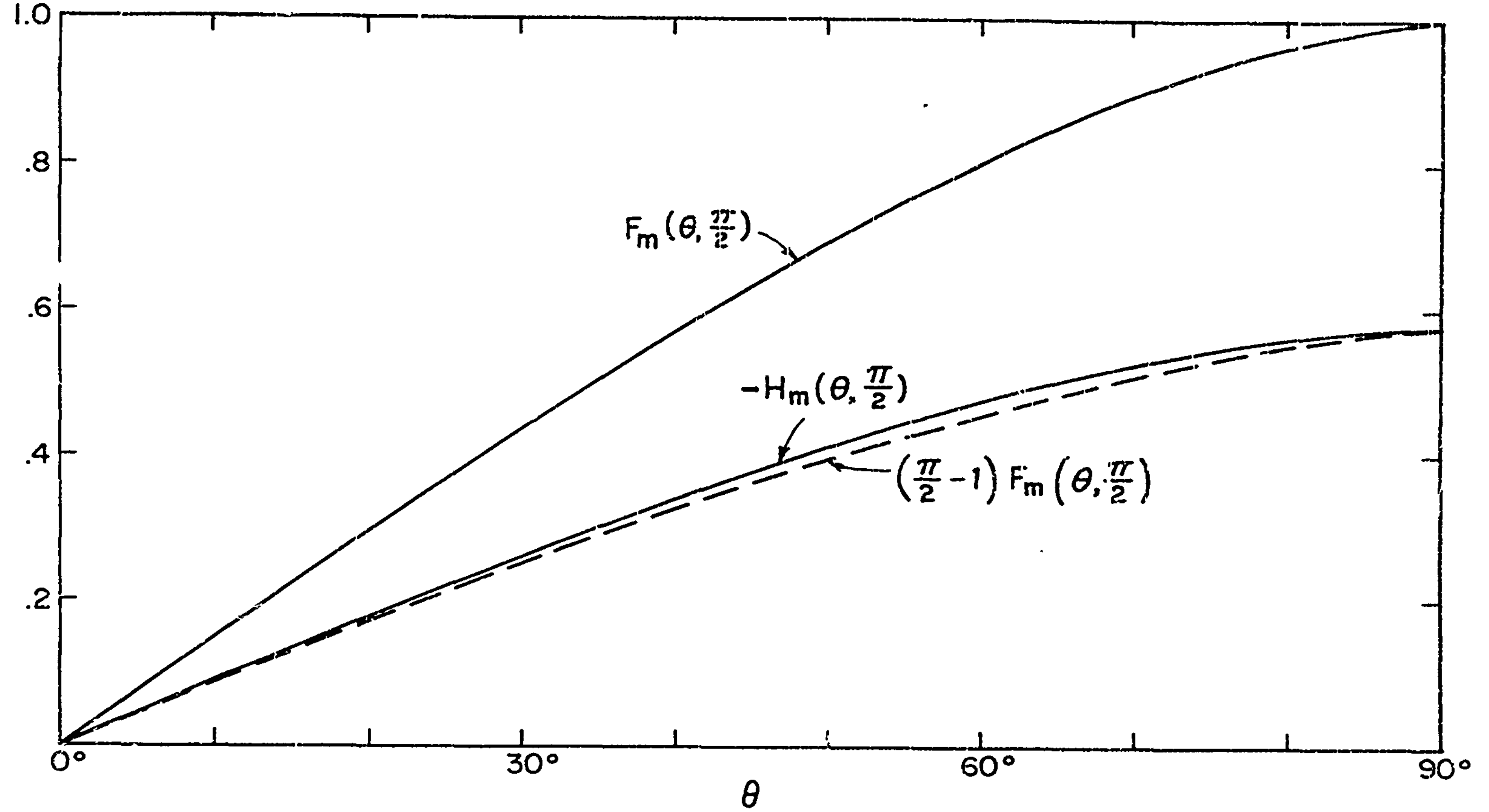

FIG. 7 THE FIELD FUNCTIONS $F_m(\theta, \frac{\pi}{2})$ AND $H_m(\theta, \frac{\pi}{2})$

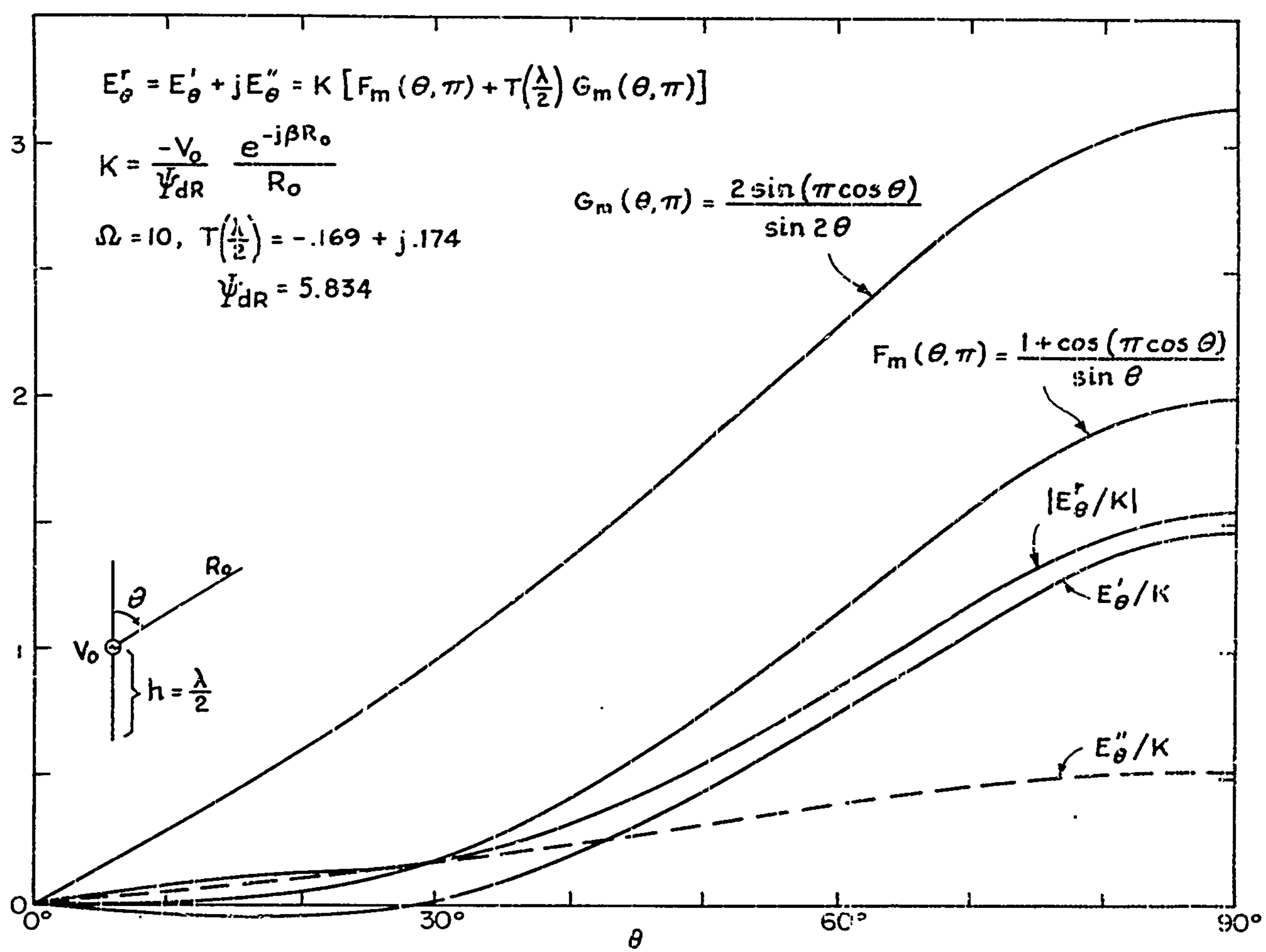

FIG. 8 VERTICAL FIELD PATTERN OF FULL-WAVE CENTER-DRIVEN ANTENNA

procedure is straightforward, since the term $\sin k(h - |z|)$ in the general expression for the current (31) is integrable when substituted in (41) and (45). However, it is not to be expected that a satisfactory approximation can be achieved if the entire term $T_k(h)(\cos kz - \cos kh)$ in (31) is simply omitted. In particular, it is essential that the total power supplied to the antenna be a good approximation even if the distribution of current along the antenna is somewhat in error. This may be achieved by a rearrangement of (31) in the following manner:

$$I(z) = [I(0)/A(0)][A(z) + B(z)] \tag{59a}$$

where

$$A(z) = \left[1 + T_k(h)\left(\frac{1 - \cos kh}{\sin kh}\right)\right]\left[\frac{\sin k(h - |z|)}{\cos kh}\right] \tag{59b}$$

and

$$B(z) = T_k(h)\left[\cos kz - 1 + \left(\frac{1 - \cos kh}{\sin kh}\right)\sin k|z|\right]. \tag{59c}$$

Note that

$$I(0) = V_o^e Y_o(k) = \left(\frac{j2\pi k V_o^e}{\omega\mu\Psi_{kr}}\right) A(0) \tag{59d}$$

where $Y_o(k)$ is the admittance. The two parts of the current, $A(z)$ and $B(z)$, are chosen so that $B(0) = 0$ and, hence, contributes only to the distribution along the antenna and not at all to the input current and the admittance, which are determined entirely by $A(0)$. The plan is to neglect $B(z)$ in the integration to determine the electromagnetic field since it contains the term that has not been integrated in closed form. The anticipated error is, therefore, that made by the omission of $B(z)$. Investigation shows that $A(z)$ alone is quite a good approximation when $\beta h \leq 3\pi/4$ and very poor when $\beta h \sim \pi$. It is particularly good when $\beta h \leq \pi/2$ and the maximum current is at the driving point. Indeed, when $\beta h = \pi/2$ and $(\alpha h)^2 \ll 1$, $B(z) = T_k(\frac{\lambda}{4})[\cos\beta z + \sin\beta|z| - 1]$. The contribution by this component of current, shown shaded in Fig. 9 when $\alpha = 0$, is seen to be quite small. Similarly, when $\beta h < 1$,

$B(z) = T_k(h)\frac{k^2h^2}{2}\left[(1 - \frac{z^2}{h^2}) - (1 - \frac{|z|}{h})\right]$. This is the very small difference

between triangular and parabolic distributions when required to have the same value at $z = 0$. If $B(z)$ is neglected in (59a) the approximate distribution is simply

$$I(z) \doteq I(0)\,\frac{\sin k(h - |z|)}{\sin kh}\,; \quad \beta h \leq 3\pi/4, \quad \alpha \leq \beta \tag{60}$$

where $I(0)$ is given in (59d).

When (60) is substituted in (41) the vector potential itself cannot be obtained in closed form. However, the derivatives required in (45) may be expressed as follows [27]:

$$B_\theta(\vec{r}) = \frac{j\mu I(0)}{4\pi\rho}\left[e^{-jkR_{1h}} + e^{-jkR_{2h}} - 2\cos kh\, e^{-jkr}\right] \tag{61}$$

$$E_\rho(\vec{r}) = \frac{j\omega\mu I(0)}{4\pi k\rho}\left[\frac{z-h}{R_{1h}}\,e^{-jkR_{1h}} + \frac{z+h}{R_{2h}}\,e^{-jkR_{2h}} - \frac{2z}{r}\cos kh\, e^{-jkr}\right] \tag{62}$$

$$E_z(\vec{r}) = \frac{-j\omega\mu I(0)}{4\pi k}\left[\frac{e^{-jkR_{1h}}}{R_{1h}} + \frac{e^{-jkR_{2h}}}{R_{2h}} - \frac{2}{r}\cos kh\, e^{-jkr}\right] \tag{63}$$

where

$$R_{1h} = \sqrt{(z-h)^2 + \rho^2}\,, \quad R_{2h} = \sqrt{(z+h)^2 + \rho^2}\,, \quad r = \sqrt{z^2+\rho^2}\,. \tag{64}$$

For the part of the current represented by (60) these expressions are accurate and give the complete electromagnetic field within distances of the cylindrical antenna comparable with the radius a. Since all distances are measured from the axis rather than the surface of the cylinder, these expressions have no meaning for $\rho < a$. They are useful particularly for determining the near field of antennas with $\beta h \leq \pi/2$.

If (61) - (63) are multiplied by $e^{j\omega t}$, the real part is the instantaneous electromagnetic field. However, the formulas so obtained permit no simple physical interpretation other than that the ends and center of the antenna may be regarded as the origins of complicated outward-traveling disturbances. A somewhat simpler picture may be obtained for the half-wave dipole in spheroidal coordinates and for the electrically short antenna in spherical coordinates.

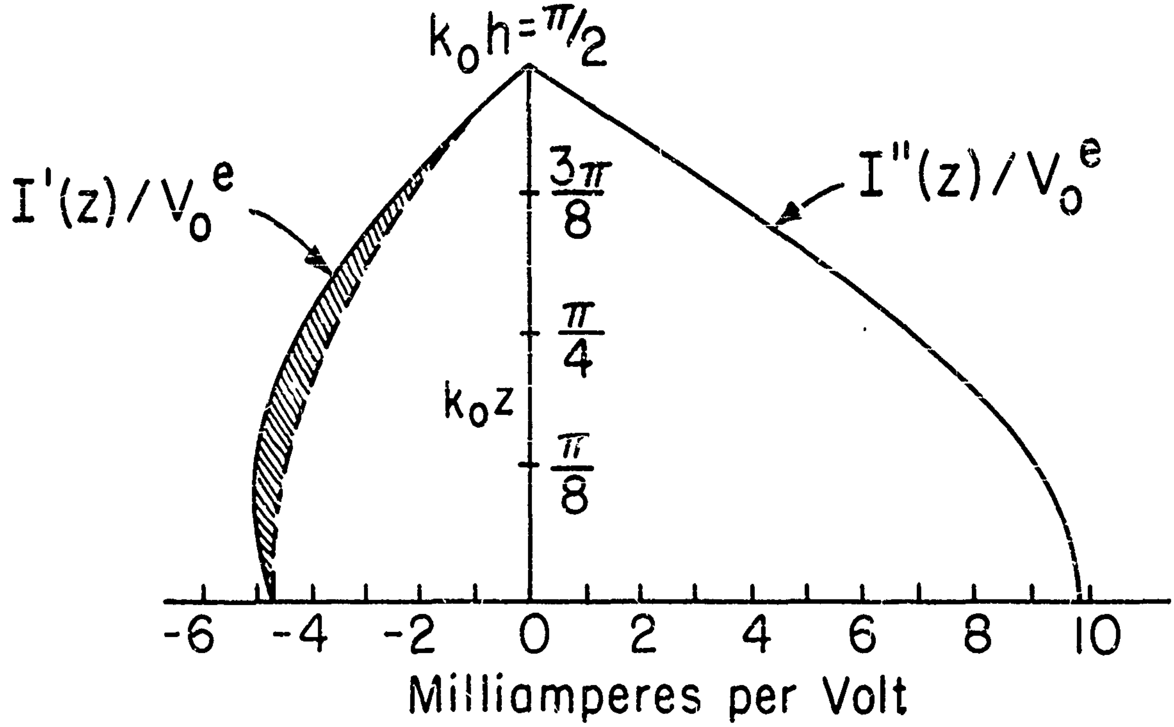

FIG. 9 CURRENT DISTRIBUTION ALONG A HALF-WAVE DIPOLE. THE BROKEN CURVE IS $I'_0 \cos k_0 z$.

3. The Half-Wave Dipole; Spheroidal Coordinates*

When $\beta h = \pi/2$ the normalized distribution of current in (60) is

$$\frac{I(z)}{I(0)} = \frac{\cos\beta z \cosh\alpha(h - |z|)}{\cosh\alpha h} + j\,\frac{\sin\beta|z|\sinh\alpha(h - |z|)}{\cosh\alpha h} \quad . \tag{65}$$

The associated distribution of charge per unit length is obtained from

$$q(z) = \mathrm{Re}\, q(z) + j\,\mathrm{Im}\, q(z) = \frac{j}{\omega}\,\frac{dI(z)}{dz} \tag{66a}$$

where in normalized form,

$$\mathrm{Re}\left[\frac{c\,q(z)}{I(0)}\right] = \frac{\beta\cos\beta z \sinh\alpha(h - |z|) - \alpha\sin\beta|z|\cosh\alpha(h - |z|)}{\beta\cosh\alpha h} \tag{66b}$$

$$\mathrm{Im}\left[\frac{c\,q(z)}{I(0)}\right] = -\left[\frac{\sin\beta|z|\cosh\alpha(h - |z|) + \alpha\cos\beta z \sinh\alpha(h - |z|)}{\beta\cosh\alpha h}\right] \cdot \tag{66c}$$

The real and imaginary parts of $I(z)/I(0)$ and $c\,q(z)/I(0)$ are shown in Fig. 10 for $\alpha h = 0$, 0.1, 0.5, 1.0, and $\pi/2$; c is the velocity of light.

Let the spheroidal coordinates k_h, k_e, and Φ be introduced where $k_e = a_e/h$ is the reciprocal of the eccentricity of a spheroid with semi-major axis a_e and with its foci at the ends of the antenna, $k_h = a_h/h$ is the corresponding quantity for an orthogonal hyperboloid of two sheets, and Φ is the azimuthal angle. These coordinates are related to the cylindrical coordinates by

$$\rho = h\sqrt{(k_e^2 - 1)(1 - k_h^2)} \qquad z = hk_hk_e\ ;$$

they are illustrated in Fig 11. Note that $1 \leqq k_e \leqq \infty$, $-1 \leqq k_h \leqq 1$. The spheroidal components of the electromagnetic field that correspond to the cylindrical components (61) - (63) are quite complicated when $\alpha \neq 0$. It is convenient to express the instantaneous value of each component as the sum of three terms as follows:

$$B_\Phi(\vec{r},t) = \sum_{i=1}^{3} B_{\Phi i}(\vec{r},t)\ , \quad E_e(\vec{r},t) = \sum_{i=1}^{3} E_{ei}(\vec{r},t)\ , \quad E_h(\vec{r},t) = \sum_{i=1}^{3} E_{hi}(\vec{r},t) \tag{67}$$

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*Much of the work in this section was carried out originally by Mr. S. T. Yu.

where

$$B_{\phi 1}(\vec{r},t) = -\mu A_1 \left[\frac{\cos(\pi k_h/2)\sin(\omega t - \pi k_e/2)}{\sqrt{(k_e^2 - 1)(1 - k_h^2)}}\right] \quad (68a)$$

$$E_{e1}(\vec{r},t) = -\zeta_a A_1 \left[\frac{\cos(\pi k_h/2)\sin(\omega t + \phi - \pi k_e/2)}{\sqrt{(k_e^2 - k_h^2)(1 - k_h^2)}}\right] \quad (68b)$$

$$E_{h1}(\vec{r},t) = \zeta_a A_1 \left[\frac{\sin(\pi k_h/2)\cos(\omega t + \phi - \pi k_e/2)}{\sqrt{(k_e^2 - k_h^2)(k_e^2 - 1)}}\right] \quad (68c)$$

$$A_1 = \frac{I(0)}{2\pi h}\cosh\left(\frac{\pi a k_h}{2\beta}\right) \exp\left(-\frac{\pi a k_e}{2\beta}\right) \quad ; \quad (68d)$$

$$B_{\Phi 2}(\vec{r},t) = -\mu A_2 \frac{\sin(\pi k_h/2)\cos(\omega t - \pi k_e/2)}{\sqrt{(k_e^2 - 1)(1 - k_h^2)}} \quad (69a)$$

$$E_{e2}(\vec{r},t) = -\zeta_a A_2 \frac{\sin(\pi k_h/2)\cos(\omega t + \phi - \pi k_e/2)}{\sqrt{(k_e^2 - k_h^2)(1 - k_h^2)}} \quad (69b)$$

$$E_{h2}(\vec{r},t) = \zeta_a A_2 \frac{\cos(\pi k_h/2)\sin(\omega t + \phi - \pi k_e/2)}{\sqrt{(k_e^2 - k_h^2)(k_e^2 - 1)}} \quad (69c)$$

$$A_2 = \frac{I(0)}{2\pi h}\sinh\left(\frac{\pi a k_h}{2\beta}\right) \exp\left(-\frac{\pi a k_e}{2\beta}\right) \quad ; \quad (69d)$$

$$B_{\Phi 3}(\vec{r},t) = \mu A_3 \frac{\cos\left(\omega t - \frac{1}{2}\pi\sqrt{k_e^2 + k_h^2 - 1}\right)}{\sqrt{(k_e^2 - 1)(1 - k_h^2)}} \quad (70a)$$

$$E_{e3}(\vec{r},t) = \zeta_a A_3 \frac{k_e\cos\left(\omega t + \phi - \frac{1}{2}\pi\sqrt{k_e^2 + k_h^2 - 1}\right)}{\sqrt{(k_e^2 + k_h^2 - 1)(k_e^2 - k_h^2)(1 - k_h^2)}} \quad (70b)$$

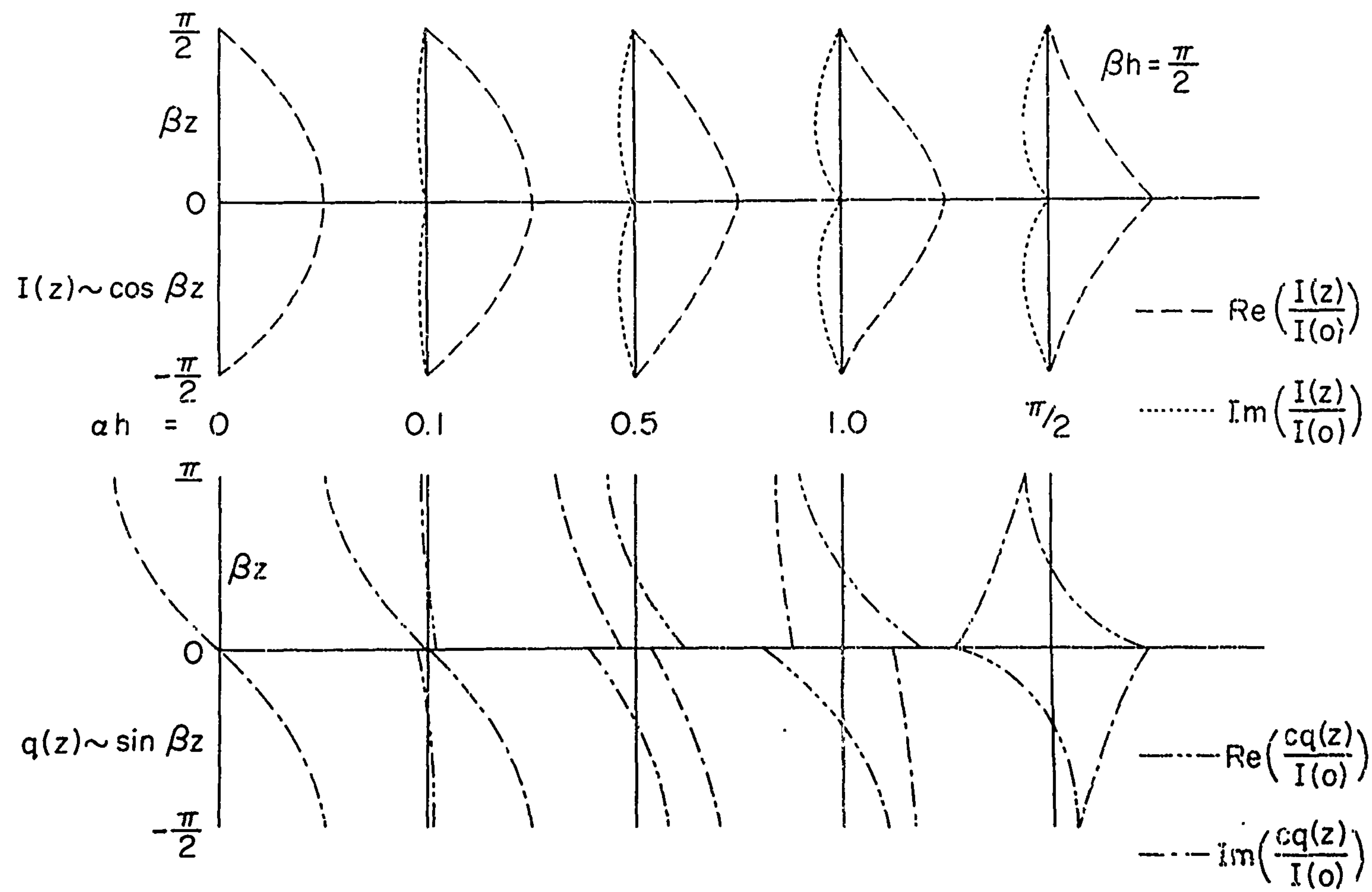

FIG. 10 NORMALIZED CURRENT AND CHARGE IN ANTENNA IN DISSIPATIVE MEDIUM. (APPROXIMATE THEORY)

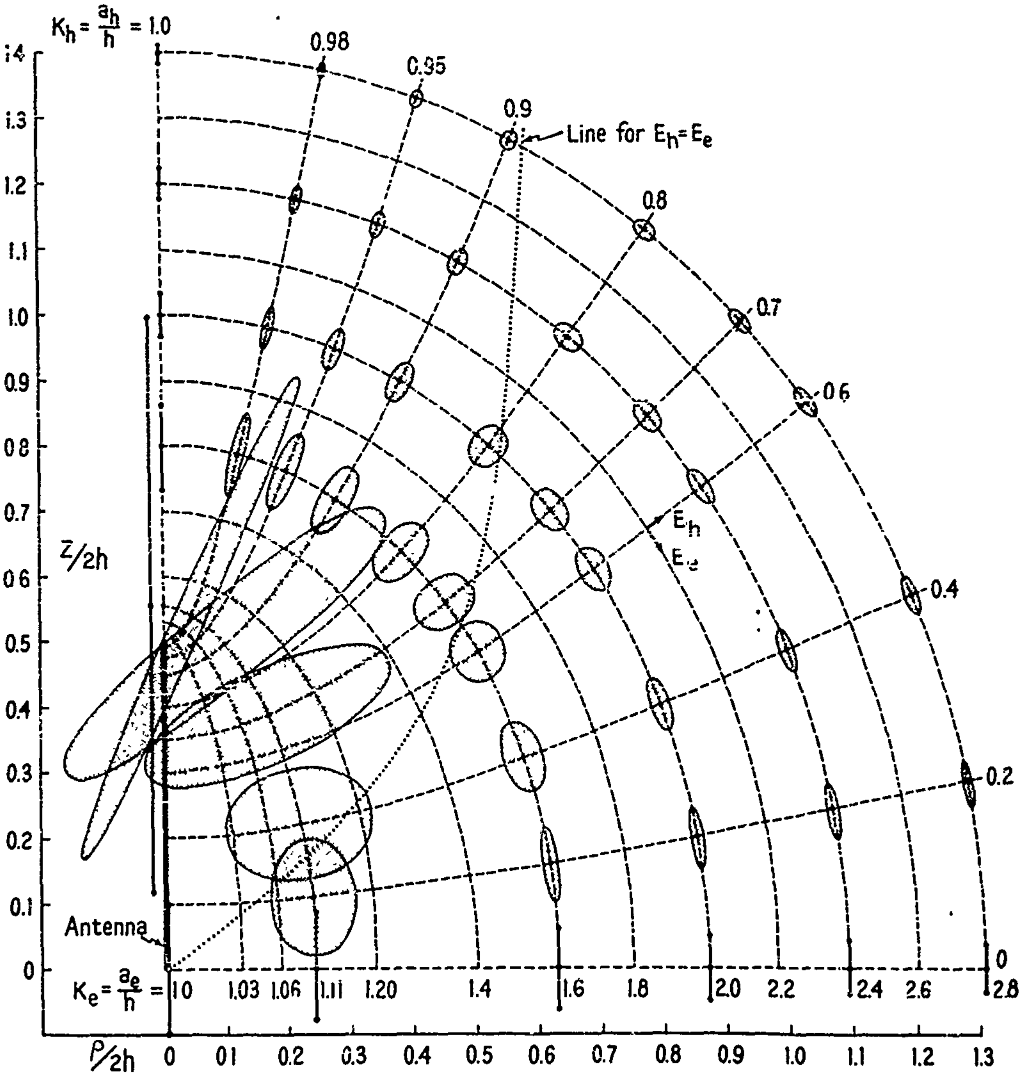

FIG. 11 ELECTRIC FIELD NEAR A HALF-WAVE ANTENNA

$$E_{h3}(\vec{r},t) = \zeta_a A_3 \frac{k_h \cos\left(\omega t + \phi - \frac{1}{2}\pi\sqrt{k_e^2 + k_h^2 - 1}\right)}{\sqrt{(k_e^2 + k_h^2 - 1)(k_e^2 - k_h^2)(k_e^2 - 1)}} \tag{70c}$$

$$A_3 = \frac{I(0)}{2\pi h} \sinh\left(\frac{\pi a}{2\beta}\right) \exp\left(-\frac{\pi a\sqrt{k_e^2 + k_h^2 - 1}}{2\beta}\right) . \tag{70d}$$

Note that $\beta h = \pi/2$, $\phi = \tan^{-1}(a/\beta)$, $\zeta_a = \omega\mu/\sqrt{a^2 + \beta^2}$.

The third part of the field as given in (70a) - (70d) may be expressed in the spherical coordinates r , Θ, and Φ in the following equivalent form:

$$B_{\Phi 3}(\vec{r},t) = \frac{\mu I(0)}{2\pi} \frac{\cos(\omega t - \beta r)}{r \sin\Theta} \sinh\left(\frac{\pi a}{2\beta}\right) \exp(-ar) \tag{71a}$$

$$E_{\Theta 3}(\vec{r},t) = \frac{\zeta_a I(0)}{2\pi} \frac{\cos(\omega t + \phi - \beta r)}{r \sin\Theta} \sinh\left(\frac{\pi a}{2\beta}\right) \exp(-ar) \tag{71b}$$

$$E_{r3}(\vec{r},t) = 0 \tag{71c}$$

These rather elaborate expressions for the electromagnetic field reduce to a simple and readily interpreted form when the attenuation of the medium is rather small. Specifically, if the following inequality is satisfied:

$$a/\beta < ah << 1 \tag{72}$$

the entire electromagnetic field of the half-wave dipole is well approximated by the first terms (68a) - (68c) in the sums in (67) with $\phi \doteq 0$ and with

$$A_1 \doteq \frac{I(0)}{2\pi h} \exp\left(-\frac{\pi a k_e}{2\beta}\right) = \frac{I(0)}{2\pi h} \exp(-a a_e) . \tag{73}$$

Except for the addition of the exponential term in the amplitude (73), the field given by (68a) - (68c) with $\phi = 0$ is like that for the same antenna in air. The components $B_\Phi(\vec{r},t)$ and $E_e(\vec{r},t)$ are in phase with each other and both are a quarter period out of phase with $E_h(\vec{r},t)$. The surfaces of constant phase are the spheroids k_e = constant and these expand outward in such a manner that the end point of the semi-major axis travels with the velocity c . The electric field at any point is elliptically polarized as shown in Fig. 11. The components $B_\Phi(\vec{r},t)$ and $E_e(\vec{r},t)$ decrease in amplitude with distance from the antenna in

a manner that approaches $1/r$ at great distances; the component $E_h(\vec{r}, t)$, on the other hand, decreases in a manner that approaches $1/r^2$. The entire field is exponentially attenuated in amplitude with the semi-major axis $a_e = hk_e$ as the variable. The instantaneous value of the outwardly directed component $S_h(\vec{r}, t)$ of the Poynting vector associated with the electromagnetic field that satisfies (72) is

$$S_h(r,t) = \frac{\zeta_e I^2(0)}{4\pi^2 h^2} \frac{\cos^2(\pi k_h/2)\exp(-\pi a k_e/\beta)}{(1-k_h^2)\sqrt{(k_e^2-k_h^2)(k_e^2-1)}} \sin^2(\omega t - \pi k_e/2) \tag{74}$$

where $\zeta_e = \omega\mu/\beta$. The time-average power transferred across a spheroidal surface with semi-major axis $a_e = hk_e$ is

$$\overline{T}_e = \frac{1}{2} I^2(0) R_o^e \; e^{-2a a_e} \tag{75}$$

where $R_o^e = 73.1$ ohms. Since terms of the order of magnitude ah have been neglected, the exponential in (75) reduces to unity when a_e approaches h, and (75) reduces to the value for air. The difference

$$T_{e1} - T_{e2} = \frac{1}{2} I^2(0) R_o^e \left[e^{-2a a_{e1}} - e^{-2a a_{e2}}\right] , \tag{76}$$

where a_{e1} and a_{e2} are the semi-major axes of two spheroids with $a_{e2} > a_{e1}$, is the power dissipated in heating the slightly conducting medium in the volume between the two surfaces. As pointed out in conjunction with (57) and (58) the correct power when the approximate distribution of current (60) with $\beta h = \pi/2$ is used is obtained with the value of $I(0)$ given by (58) instead of the correct value given by (32). Alternatively, the correct power is also obtained if $I(0)$ is obtained from (32) and the correct input resistant R_o is substituted in (76) for the radiation resistance R_o^e as computed with the approximate current.

When a is not sufficiently small to satisfy (72) with $\beta h = \pi/2$ the complete expressions (67) with (68a) - (70d) must be used for the electromagnetic field. These cannot be combined to give a simple picture of outward traveling surfaces of constant phase at all points outside the half-wave dipole. It is not even convenient to combine the first and second parts of the field into a spheroidal wave and treat the third part in the forms (71a) - (71c) as a

spherical wave originating at the center since components of both the second and third parts become infinite along the z axis and only their sum is finite. However, if a relatively small region in the vicinity of the antenna is excluded, the complete field can be approximated by spheroidal waves similar to those which obtain when the attenuation constant is small. In effect, the spherical wave originating at the center of the antenna must be approximated by a spheroidal wave. It is readily verified that if the quantity

$$d = 1 - \sqrt{1 - (1 - k_h^2)/k_e^2} \tag{77}$$

is small enough so that it may be neglected completely, the third part of the field given by (70a) - (70d) may be approximated by

$$B_{\Phi 3}(\vec{r}, t) \doteq \mu A_3 \frac{\cos(\omega t - \pi k_e/2)}{\sqrt{(k_e^2 - 1)(1 - k_h^2)}} \tag{78a}$$

$$E_{e3}(\vec{r}, t) \doteq \zeta_a A_3 \frac{\cos(\omega t + \phi - \pi k_e/2)}{\sqrt{(k_e^2 - k_h^2)(1 - k_h^2)}} \tag{78b}$$

$$E_{h3}(\vec{r}, t) \doteq \zeta_a A_3 \frac{k_h \cos(\omega t + \phi - \pi k_e/2}{k_e \sqrt{(k_e^2 - k_h^2)(k_e^2 - 1)}} \tag{78c}$$

$$A_3 = \frac{I(0)}{2\pi h} \sinh\left(\frac{\pi a}{2\beta}\right) \exp\left(-\frac{\pi a k_e}{2\beta}\right) . \tag{78d}$$

It is to be noted that over a considerable range near the z axis k_h is very near one so that $d \doteq 0$. In the vicinity of the equatorial plane where k_h is very small, the excluded range is determined by the magnitude of k_e, it is indicated in Fig. 12. Specifically, for example, $d \leq 0.1$ when $(1 - k_h^2)/k_e^2 \leq 0.19$.

With the approximate expressions (78a) - (78d) and (68a) - (69d) the following combinations may be made:

$$B_{\Phi 2}(\vec{r},t) + B_{\Phi 3}(\vec{r},t) = -\frac{\mu I(0)}{2\pi h} \frac{D_2 \cos(\omega t - \pi k_e/2)}{\sqrt{(k_e^2 - 1)(1 - k_h^2)}} \exp\left(-\frac{\pi a k_e}{2\beta}\right) \tag{79a}$$

$$E_{e2}(\vec{r},t) + E_{e3}(\vec{r},t) = -\frac{\zeta_a I(0)}{2\pi h} \frac{D_2 \cos(\omega t + \phi - \pi k_e/2)}{\sqrt{(k_e^2 - k_h^2)(1 - k_h^2)}} \exp\left(-\frac{\pi a k_e}{2\beta}\right) \tag{79b}$$

$$E_{h1}(\vec{r},t) + E_{h3}(\vec{r},t) = \frac{\zeta_a I(0)}{2\pi h} \frac{F_1 \cos(\omega t + \phi - \pi k_e/2)}{\sqrt{(k_e^2 - k_h^2)(k_e^2 - 1)}} \exp\left(-\frac{\pi a k_e}{2\beta}\right) \tag{79c}$$

where

$$D_2 = \sinh\left(\frac{\pi a k_h}{2\beta}\right) \sin\left(\frac{\pi k_h}{2}\right) - \sinh\left(\frac{\pi a}{2\beta}\right) \tag{79d}$$

$$F_1 = \cosh\left(\frac{\pi a k_h}{2\beta}\right) \sin\left(\frac{\pi k_h}{2}\right) + \frac{k_h}{k_e} \sinh\left(\frac{\pi a}{2\beta}\right) . \tag{79e}$$

With the additional notation,

$$D_1 = \cosh\left(\frac{\pi a k_h}{2\beta}\right) \cos\left(\frac{\pi k_h}{2}\right) \tag{80a}$$

$$F_2 = \sinh\left(\frac{\pi a k_h}{2\beta}\right) \cos\left(\frac{\pi k_h}{2}\right) \tag{80b}$$

the complete field may be obtained by combining (79a) and (79b) respectively with (68a) and (68b), and (79c) with (69c). The results are

$$B_{\Phi}(\vec{r},t) = -\frac{\mu I(0)}{2\pi h} \frac{D \sin(\omega t + \Delta - \pi k_e/2)}{\sqrt{(k_e^2 - 1)(1 - k_h^2)}} \exp\left(-\frac{\pi a k_e}{2\beta}\right) \tag{81a}$$

$$E_e(\vec{r},t) = -\frac{\zeta_a I(0)}{2\pi h} \frac{D \sin(\omega t + \phi + \Delta - \pi k_e/2)}{\sqrt{(k_e^2 - k_h^2)(1 - k_h^2)}} \exp\left(-\frac{\pi a k_e}{2\beta}\right) \tag{81b}$$

$$E_h(\vec{r},t) = \frac{\zeta_a I(0)}{2\pi h} \frac{F \cos(\omega t + \phi + \Psi - \pi k_e/2)}{\sqrt{(k_e^2 - k_h^2)(k_e^2 - 1)}} \exp\left(-\frac{\pi a k_e}{2\beta}\right) \tag{81c}$$

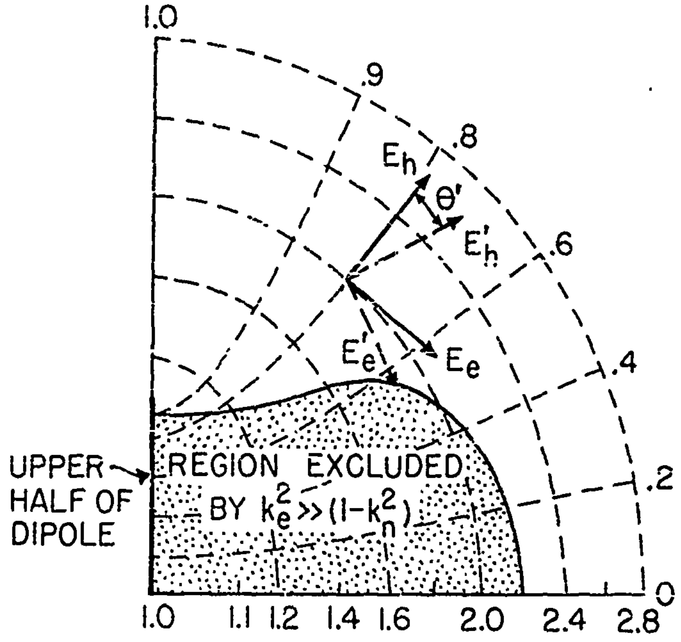

FIG. 12 REGION EXCLUDED BY $k_e^2 \gg (1-k_n^2)$; ROTATION OF PRINCIPAL AXES THROUGH ANGLE θ'

where

$$D = \sqrt{D_1^2 + D_2^2}\ , \qquad \Delta = \tan^{-1}(D_2/D_1) \tag{82a}$$

and

$$F = \sqrt{F_1^2 + F_2^2}\ , \qquad \Psi = -\tan^{-1}(F_2/F_1)\ . \tag{82b}$$

The formulas (81a) - (81c) for the spheroidal components of the field of a half-wave dipole ($\beta h = \pi/2$) in a dissipative medium without restriction on the conductivity are substantially like the formulas for the same antenna in a medium that is only slightly conducting. However, the formulas for the unrestricted medium are not useful in a region near the antenna where the approximation $d \doteq 0$ is not a good approximation. In addition, they are more complicated owing to the appearance of the different phases Δ and Ψ in the expressions for the mutually perpendicular components $E_e(\vec{r},t)$ and $E_h(\vec{r},t)$ of the electric field.

The time-average component of the Poynting vector perpendicular to the spheroidal surfaces k_e = constant is

$$\overline{S}_h(r) = \frac{I^2(0)}{8\pi^2 h^2}\ \frac{D^2 \cos\phi \exp(-\pi a k_e/\beta)}{(1-k_h^2)\sqrt{(k_e^2 - k_h^2)(k_e^2 - 1)}}\ . \tag{83}$$

The total time-average power transferred across a spheroidal surface for which $k_e \geq 2.2$ is

$$\overline{T}_e = \frac{\zeta_e I^2(0) \exp(-\pi a k_e/\beta)}{4\pi(1 + a^2/\beta^2)} \int_{-1}^{1} \frac{D^2}{1 - k_h^2}\, dk_h \tag{84}$$

where $\zeta_e = \omega\mu/\beta$. Owing to the complicated form of D as a function of k_h, the integral in (84) has not been evaluated.

The general formulas (81a) - (81c) indicate that the electromagnetic field of a half-wave dipole in an arbitrary dissipative medium may be interpreted in terms of expanding spheroidal surfaces except very near the antenna where more complicated conditions exist. These surfaces expand with foci

at the ends of the antenna and with an outward phase velocity along the z axis that is given by

$$v_p = \omega/\beta \tag{85}$$

where β is the real part of the complex propagation constant k as defined in (19). However, although the phase of $B_\Phi(\vec{r},t)$, $E_e(\vec{r},t)$, or $E_h(\vec{r},t)$ remains constant* as each spheroid moves out along a hyperbola defined by a particular value of k_h, the actual phases at different points along such a spheroid are not the same. That is, the expanding spheroids are surfaces of constant phase, but the constant value on each spheroid is a function of k_h. Thus, respectively, for $B_\Phi(\vec{r},t)$, $E_e(\vec{r},t)$, and $E_h(\vec{r},t)$ the phases, $(\omega t + \Delta - \pi k_e/2)$, $(\omega t + \phi + \Delta - \pi k_e/2)$, and $(\omega t + \phi + \Psi - \pi k_e/2)$ remain constant* as t and k_e increase together. At each point on a given spheroid k_e = constant, the phase of $E_e(\vec{r},t)$ differs from that of $B_\Phi(\vec{r},t)$ by the constant angle $\phi = \tan^{-1}(\alpha/\beta)$. The phases of the two mutually perpendicular components of the electric field differ by the angle $(\Delta - \Psi)$ which is a function of k_h.

In a medium that is dissipationless or has very low attenuation, the electric field is elliptically polarized with principal axes tangent and perpendicular to the spheroidal surfaces of constant phase as shown in Fig. 12. In the dissipative medium with α unrestricted, the principal axes of the ellipses described by the electric vector are rotated from these tangents and perpendiculars by an angle θ' that is a function of k_h. The polarization ellipses for the electric field and their angle of inclination with respect to the direction of $E_h(\vec{r},t)$ may be determined if the time is eliminated between the expressions for $E_e(\vec{r},t)$ and $E_h(\vec{r},t)$ as given in (81b) and (81c). Let these formulas be expressed as follows:

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*This is strictly true for $B_\Phi(\vec{r},t)$ and $E_e(\vec{r},t)$, since k_e does not occur in the phase term Δ. It is not actually true for $E_h(\vec{r},t)$ owing to the occurrence of k_e in the term $(k_h/k_e)\sinh(\pi\alpha/2\beta)$ in F_1 which contributes to the phase in Ψ. However, this term is small, except quite near the ends of the antenna, so that the general picture is not altered significantly.

$$E_e(\vec{r},t) = E_e \sin(\Omega + \Delta) \tag{86}$$

$$E_h(\vec{r},t) = E_h \cos(\Omega + \Psi) \tag{87}$$

where

$$E_e = -\frac{\zeta_a I(0)}{2\pi h} \frac{D \exp(-\pi a k_e/2\beta)}{\sqrt{(k_e^2 - k_h^2)(1 - k_h^2)}} \tag{88}$$

$$E_h = \frac{\zeta_a I(0)}{2\pi h} \frac{F \exp(-\pi a k_e/2\beta)}{\sqrt{(k_e^2 - k_h^2)(k_e^2 - 1)}} \tag{89}$$

and

$$\Omega = (\omega t + \phi - \pi k_e/2) \ . \tag{90}$$

If (86) and (87) are solved for $\sin \Omega$ and $\cos \Omega$, and these quantities are then squared and added to equal one, the following equation is obtained:

$$\frac{E_h^2(\vec{r},t)}{E_h^2} + \frac{E_e^2(\vec{r},t)}{E_e^2} + \frac{2E_h(\vec{r},t)\, E_e(\vec{r},t)}{E_h E_e} \sin(\Psi - \Delta) = \cos^2(\Psi - \Delta) \tag{91}$$

This is the equation of an ellipse that has its principal axes rotated with respect to the directions of E_h and E_e. Let the direction of E_h' be rotated from E_h by an angle θ' as shown in Fig. 12. The direction of E_e' is then rotated by the same angle with respect to E_e. The new components E_h' and E_e' are related to the old ones by the formulas

$$E_h = E_h' \cos\theta' - E_e' \sin\theta' \tag{92a}$$

$$E_e = E_h' \sin\theta' + E_e' \cos\theta' \ . \tag{92b}$$

If the angle θ' is chosen so that

$$\tan 2\theta' = \frac{2E_h E_e}{E_e^2 - E_h^2} \sin(\Psi - \Delta) \ , \tag{93}$$

the new equation is

$$\left[\frac{E_h'(r,t)}{E_h'}\right]^2 + \left[\frac{E_e'(r,t)}{E_e'}\right]^2 = \cos^2(\Psi - \Delta) \tag{94}$$

where

$$\left(\frac{1}{E_h'}\right)^2 = \frac{\cos^2\theta'}{E_h^2} + \frac{2\sin(\Psi-\Delta)\sin\theta'\cos\theta'}{E_h E_e} + \frac{\sin^2\theta'}{E_e^2} \tag{95a}$$

$$\left(\frac{1}{E_e'}\right)^2 = \frac{\sin^2\theta'}{E_h^2} + \frac{2\sin(\Psi-\Delta)\sin\theta'\cos\theta'}{E_h E_e} + \frac{\cos^2\theta'}{E_e^2} \quad . \tag{95b}$$

The equation (94) is that of an ellipse with semi-principal axes $E_h' \cos(\Psi - \Delta)$ and $E_e' \cos(\Psi - \Delta)$. Note that when $\Psi = \Delta$, as in a dissipationless medium, $\theta' = 0$, $E_h' = E_h$, $E_e' = E_e$. It can be argued from symmetry or determined directly from the formulas, that $\theta' = 0$ when $k_h = 0$, 1 ; it follows that the electric field is still linearly polarized parallel to the antenna both along the z axis and in the equatorial plane just as when immersed in air.

4. The Electrically Short Antenna

As pointed out following (59a) - (59d), the approximate distribution of current (60) in the form

$$I(z) = I(0)\ (1 - |z|/h) \tag{96}$$

for the electrically short antenna ($\beta h \leq 0.3$, $\alpha h \leq 0.3$) is a particularly good approximation since the omitted term, $B(z) = T_k(h)\frac{k^2h^2}{2}\left[\left(1 - \frac{z^2}{h^2}\right) - \left(1 - \frac{|z|}{h}\right)\right]$ is very small. The input current is to be determined from the more accurate formula (34) . Since (96) is a special case of (60) with kh small in its real and imaginary parts, it follows that the rigorous expressions (61) - (63) for the electromagnetic field of the distribution (60) apply. They may be converted into a more common but also slightly more restricted form by a series expansion in powers of the small quantity kh and the quantity h/r , <u>which must also be assumed small</u> . The distance $r = \sqrt{\rho^2 + z^2}$ is measured from the center of the dipole to the point where the field is calculated. The approximations include

$$R_{1h} \doteq r - h\cos\theta + (h\sin\theta)^2/2r \tag{97a}$$

$$R_{2h} \doteq r + h\cos\theta + (h\sin\theta)^2/2r \tag{97b}$$

$$\cos kh \doteq 1 - k^2h^2/2 \quad . \tag{97c}$$

With these approximations the complete electromagnetic field in the spherical coordinates r , θ , Φ has the familiar form

$$B_\Phi(r) \doteq \frac{j\mu I(0)h}{4\pi}\left[\frac{k}{r} - \frac{j}{r^2}\right] e^{-jkr}\sin\theta \tag{98a}$$

$$E_\theta(r) \doteq \frac{j\omega\mu I(0)h}{4\pi k}\left[\frac{k}{r} - \frac{j}{r^2} - \frac{1}{kr^3}\right] e^{-jkr}\sin\theta \tag{98b}$$

$$E_r(r) \doteq \frac{\omega\mu I(0)h}{4\pi k}\left[\frac{2}{r^2} - \frac{j2}{kr^3}\right] e^{-jkr}\sin\theta \quad . \tag{98c}$$

Note that $\omega\mu/k = \sqrt{\mu/\xi} = \zeta$. In this derivation terms of the order of magnitude $|kh|^4$ and h^3/r^3 and higher powers have been neglected. This means that the field given in (97a) - (97c) is not valid in the immediate vicinity of the antenna as are the more general expressions (61) - (63). If desired, the above formulas may be expressed in terms of the equivalent electric moment p_z with the relation

$$j\omega p_z = \int_{-h}^{h} I(z)\,dz = I(0)h \quad . \tag{99}$$

For a short dipole in air, $k = k_o$ is real; $\omega\mu/k = \zeta_o$. The instantaneous field is obtained from the real parts of (98a) - (98c) when expressed in polar form and multiplied by $e^{j\omega t}$. Specifically,

$$B_\Phi(\vec{r}, t) = \frac{\mu_o I(0)h}{4\pi}\,\frac{\sqrt{1 + k_o^2 r^2}}{r^2}\cos(\omega t - k_o r + \tan^{-1} k_o r)\sin\theta \tag{100a}$$

$$E_\theta(\vec{r}, t) = \frac{\zeta_o I(0)h}{4\pi}\,\frac{\sqrt{1 - k_o^2r^2 + k_o^4r^4}}{k_o r^3}\cos[\omega t - k_o r + \tan^{-1}(k_o r - 1/k_o r)]\sin\theta \tag{100b}$$

$$E_r(\vec{r},t) = \frac{\zeta_o I(0)h}{2\pi} \frac{\sqrt{1+k_o^2 r^2}}{k_o r^3} \cos(\omega t - k_o r + \cot^{-1} k_o r)\cos\theta \quad \cdot \tag{100c}$$

These expressions do not in general permit the simple interpretation of spherical surfaces of constant phase that travel radially outward with a definite phase velocity as is true in terms of spheroidal waves of the half-wave dipole in air. Each component may be so interpreted, but the phase velocities of all three components are different until $k_o r$ becomes sufficiently great in the radiation zone so that $k_o^2 r^2 >> 1$, $\tan^{-1} k_o r \doteq \pi/2$, $\cot^{-1} k_o r \doteq 0$. The radiation-zone field is

$$B_\Phi^r(\vec{r},t) = E_\theta^r(\vec{r},t)/c = \frac{\mu_o I(0)k_o h}{4\pi r} \sin(\omega t - k_o r)\sin\theta \tag{101a}$$

$$E_r^r(\vec{r},t) = \frac{\zeta_o I(0)k_o h}{2\pi r^2} \cos(\omega t - k_o r)\cos\theta \doteq 0 \quad . \tag{101b}$$

These formulas represent a true spherical wave. All components expand with the same phase velocity c; $B_\Phi^r(\vec{r},t)$ and $E_\theta^r(\vec{r},t)$ are in phase with each other and a quarter period out of phase with $E_r^r(\vec{r},t)$. $B_\Phi^r(\vec{r},t)$ and $E_\theta^r(\vec{r},t)$ decrease as $1/r$; $E_r^r(\vec{r},t)$ decreases as $1/r^2$ so that when r is large this component is insignificant as indicated in (101b).

The radial component of the complex Poynting vector for the field in (100a) - (100c) is

$$S_r(\vec{r}) = \frac{|I(0)|^2 k_o^2 h^2 \zeta_o}{32\pi^2} \left[\frac{1}{r^2} - \frac{j}{k_o^3 r^5}\right] \sin^2\theta \ , \tag{102}$$

and the total time-average power radiated is

$$\overline{T}_e = \mathrm{Re}\int_0^{2\pi} d\Phi \int_0^{\pi} S_r(r)\, r^2 \sin\theta\, d\theta = \frac{1}{2} I^2(0)\, R_o^e \tag{103}$$

where the radiation resistance is

$$R_o^e = \frac{\zeta_o k_o^2 h^2}{6\pi} = 20 k_o^2 h^2 \text{ ohms} \qquad . \tag{104}$$

Since the approximate current (96) was used to determine the power radiated instead of the more accurate current (34), R_o^e in (104) is only an approximation of the actual input resistance obtained from (35). The more accurate value has as its leading term,

$$R_o(k_o) = \frac{\zeta_o k_o^2 h^2}{6\pi} \frac{\Psi_{dr}}{\Omega - 3} \tag{105}$$

where $\Psi_{dr} = 2(\ln \frac{h}{a} - 1)$ and $\Omega = 2\ln(2h/a)$. When $\Omega = 10$, $h/a \doteq 75$, the ratio $\Psi_{dr}/(\Omega - 3) = 0.95$, so that (104) is in error by about 5 percent. The error decreases as h/a increases.

When the medium is dissipative with $k = \beta - j\alpha$, the components of the field may be expressed as follows:

$$B_\phi(\vec{r}) = \frac{\mu I(0)h}{4\pi} \left[\frac{j\beta}{r} + \frac{\alpha}{r} + \frac{1}{r^2}\right] e^{-\alpha r} e^{-j\beta r} \sin\theta \tag{106a}$$

$$E_\theta(\vec{r}) = \frac{\omega\mu I(0)h}{4\pi} \left[j\left(\frac{1}{r} + \frac{\alpha}{(\beta^2+\alpha^2)r^2} - \frac{\beta^2-\alpha^2}{(\beta^2+\alpha^2)^2 r^3}\right) + \frac{\beta}{(\beta^2+\alpha^2)r^2} + \frac{2\alpha\beta}{(\beta^2+\alpha^2)^2 r^3}\right] e^{-\alpha r} e^{-j\beta r} \sin\theta \tag{106b}$$

$$E_r(\vec{r}) = \frac{\omega\mu I(0)h}{2\pi} \left[\frac{\beta}{(\beta^2+\alpha^2)r^2} + \frac{2\alpha\beta}{(\beta^2+\alpha^2)^2 r^3} + j\left(\frac{\alpha}{(\beta^2+\alpha^2)\, r^2} - \frac{\beta^2-\alpha^2}{(\beta^2+\alpha^2)^2 r^3}\right)\right] e^{-\alpha r} e^{-j\beta r} \cos\theta \tag{106c}$$

The real part of the complex Poynting vector is

$$\text{Re } S_r(r) = \frac{|I(0)|^2 \beta^2 h^2 \zeta_e}{32\pi^2 r^2} \left(1 + \frac{\alpha}{\beta} L\right) e^{-2\alpha r} \tag{107}$$

where

$$L = \frac{2}{(1 + \alpha^2/\beta^2)\beta r} + \frac{4\alpha/\beta}{(1 + \alpha^2/\beta^2)^2 \beta^2 r^2} + \frac{2}{(1 + \alpha^2/\beta^2)^2 \beta^3 r^3} \tag{108}$$

and where $\zeta_e = \omega\mu/\beta$. The total time-average power transferred outward across a spherical surface of radius r is the integral of (107) over the surface of the sphere. The result is

$$\overline{T}_e = \frac{|I(0)|^2 \beta^2 h^2 \zeta_e}{12\pi} \left(1 + \frac{\alpha}{\beta} L\right) e^{-2\alpha r} \quad . \tag{109}$$

As in the case of (103) this formula is an approximation that is quite good only for extremely thin antennas. It may be corrected to apply to antennas of somewhat greater thickness by multiplying (109) by the factor $\Psi_{dr}/(\Omega - 3)$. It is important to note that (109) cannot be used to determine the radiation resistance of the short antenna in a dissipative medium since it is not possible to reduce the spherical surface across which the power T_e is transferred to an envelope that encloses only the antenna and no part of the medium. The smallest spherical surface that contains the antenna has the radius $r = h$, but, since in the transformation from cylindrical to spherical coordinates, terms of the order of magnitude $(h/r)^3$ have been neglected in comparison with unity, (109) is meaningful only when $r > h$. Since a sphere of radius $r > h$ contains a significant part of the dissipative medium, the power transferred across its surface is not the total power radiated from the dipole. Note in particular, that the limit $r = 0$ is meaningless.

The actual power supplied to the antenna may be obtained from the driving-point admittance $Y_o(k)$ in (34) in the form $T_e = \frac{1}{2} V_o^{e2} G_o(k) = \frac{1}{2}|I(0)|^2 R_o(k)$. Since the complete expressions for $G_o(k)$ and $B_o(k)$ are rather long, it will serve for purposes of illustration to consider the two special cases, $\alpha^2 \ll \beta^2$ and $\alpha = \beta$, for which the formulas are much simpler. The resistance $R_o(k)$ is given by

$$R_o(k) = \frac{G_o(k)}{G_o^2(k) + B_o^2(k)} \quad . \tag{110}$$

When $\alpha^2 << \beta^2$, $G_o^2(k) << B_o^2(k)$ so that

$$R_c(k) \doteq \frac{G_o(k)}{B_o^2(k)} = \frac{\zeta_e \beta^2 h^2 \Psi_{dr}}{6\pi(\Omega - 3)} \left[1 + \frac{6\alpha(\Omega - 3)}{\beta^2 h}\right] \tag{111}$$

if only the leading terms are retained. When $\alpha = \beta$, $G_o^2(k) >> B_o^2(k)$, so that from (40)

$$R_o(k) \doteq \frac{1}{G_o(k)} = \frac{\zeta_e \Psi_{dr}}{4\pi} \frac{1}{\beta h} \quad . \tag{112}$$

The power per unit input current, P_r, that is dissipated within a sphere of radius $r > h$ is defined as follows in terms of the power $\overline{T}_e$ supplied to the antenna and the power $\overline{T}_{er}$ transferred outward to the region beyond the sphere:

$$P_r = \frac{2(\overline{T}_e - \overline{T}_{er})}{|I(0)|^2} = R_o(k) - \frac{\beta^2 h^2 \zeta_e \Psi_{dr}}{6\pi(\Omega - 3)} (1 + \frac{\alpha}{\beta} L) e^{-2\alpha r} \quad . \tag{113}$$

When $\alpha^2 << \beta^2$ this becomes

$$P_r = \frac{\zeta_e \beta^2 h^2 \Psi_{dr}}{6\pi(\Omega - 3)} \frac{\alpha}{\beta} \left[\frac{6(\Omega - 3)}{\beta h} - L e^{-2\alpha r}\right] \tag{114a}$$

where

$$L = \frac{z}{\beta r} + \frac{z}{\beta^3 r^3} \quad . \tag{114b}$$

When $\alpha = \beta$,

$$P_r = \frac{\zeta_e \Psi_{dr}}{4\pi} \left[\frac{1}{\beta h} - \frac{\beta^2 h^2}{3(\Omega - 3)} (1 + L) e^{-2\beta r}\right] \tag{115a}$$

where

$$L = (\frac{1}{\beta r} + \frac{1}{\beta^2 r^2} + \frac{1}{2\beta^3 r^3}) \quad . \tag{115b}$$

It is instructive to consider a numerical example. Let the power dissipated within the radian sphere $\beta r = 1$ be determined for an antenna with $\beta h = 0.3$ and $h/a \doteq 75$. When $\alpha^2 << \beta^2$, $e^{-2\alpha r} = e^{-2\alpha/\beta} \doteq 1 - 2\alpha/\beta$; $L \doteq 4$.

With these values, the power supplied to the antenna per unit current is

$$2\overline{T}_e/|I(0)|^2 = R_o(k) = 1.71\,(1 + 140\,\alpha/\beta)\text{watts/amp}^2; \tag{116a}$$

the power transferred beyond the radian sphere per unit current is

$$2\overline{T}_{er}/|I(0)|^2 = 1.71\,(1 + 2\alpha/\beta)\ \text{watts/amp}^2\ ; \tag{116b}$$

and the power dissipated as heat in the medium within the radian sphere is

$$P_r = 236\ \alpha/\beta\ \ \text{watts/amp}^2\ . \tag{116c}$$

When $\alpha = \beta$, the corresponding values are

$$2\overline{T}e/|I(0)|^2 = R_o(k) = 1990.5\ \text{watts/amp}^2 \tag{117a}$$

$$2\overline{T}_{er}/|I(0)|^2 = 0.81\ \text{watts/amp}^2 \tag{117b}$$

$$P_r = 1989.7\ \text{watts/amp}^2 \tag{117c}$$

From these numerical results it is clear that when α/β is as small as 10^{-3} or smaller, 90 percent or more of the power supplied to the antenna is transferred beyond the radian sphere. When α/β is no greater than 0.1, only about 8 percent is dissipated outside; 92 percent is used to heat the medium inside the radian sphere. When $\alpha = \beta$ virtually all of the power is dissipated as heat within the radian sphere. The fraction transferred beyond it is only about 0.04 percent. Note that when $\alpha = \beta$ the input susceptance of the antenna is small, so that virtually the entire impedance is resistive. The antenna effectively does not radiate, but acts like a pair of electrodes with very small surface area--hence the rather high resistance.

Conclusion

The general problem of a cylindrical antenna immersed in a dissipative medium has been formulated in a manner that permits the determination not only of the distribution of current and the admittance, but also of the electromagnetic field. The analytical procedure is approximate but quantitatively sufficiently accurate to be of practical value. It may be extended to treat coupled antennas in a dissipative medium.

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